Flood Prevention by Optimal Dike Heightening

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Abstract

Dike height optimization is of major importance to the Netherlands since a large part of the country is below sea level and high water levels in rivers may cause floods. In this paper we propose a cost-benefit analysis to determine optimal dike heights. We improve the model proposed by Van Dantzig (1956) after a devastating flood in the Netherlands in 1953. For one important class of the dike investment cost function we show that there is periodic solution that satisfies the first order conditions, and this solution can be easily computed. For the general case we propose a Dynamic Programming approach. Numerical results suggest that the current dike height standards in the Netherlands are too low. The Dutch government has already reserved extra funds to realize these higher standards, and it is expected that the standards in the Dutch Water Act will be adapted in the near future.
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1 Introduction

Protection against flooding is an important issue in the Netherlands, since 55% of this country is susceptible to flood risk. Each year the government spends 1 billion euros on protection by dikes, structures and dunes. In total there are 3500 km of dikes in the Netherlands. In 1953 a severe flooding disaster occurred in the south-western parts of the Netherlands, killing 2000 people and causing immense economic damage. In 1995 there was again a critical situation, this time along the major rivers of the Rhine and Meuse, forcing 200,000 people to be evacuated, but fortunately there was no serious flooding.

Protection against flooding is an important issue worldwide. There are many deltas that must fight against the water. In 2005, serious flooding in and around New Orleans saw some 1500 people killed, with extremely serious damage. In other countries, like Bangladesh, serious flooding is an almost yearly phenomenon. In 2010 flooding has been a serious issue in several areas, of which Pakistan was the most serious. Syvitski et al. (2009) gives an analysis of 33 important deltas, and it was found that 85% of these deltas had experienced severe flooding in the past decade. Close to half a billion people live on or near deltas, often in megacities. They estimate that the delta surface that is vulnerable to flooding could increase by 50% under the currently projected values for sea level rise in the 21st century, due to global climate change. This is often aggravated by the decline of the ground level by the draining of marshlands and pumping fresh water out of deep wells or the extraction of oil and gas. The renowned Stern Review (Stern (2006)) offers an analysis of the economic consequences of the sea level rise, of which flood protection and damage are certainly aspects.

In this paper we carry out an economic cost-benefit analysis for the efficient protection against flooding in the Netherlands. The costs mainly consist of investment costs to heighten dikes, and the benefit is more safety and thus less (expected) damage costs due to flooding. We show that for certain choices of the investment cost function, there is a periodic solution that satisfies the first order conditions, and this solution can be stated explicitly. We propose a Dynamic Programming approach to find the optimal solution if other investment cost functions are used.

After the flood disaster in the Netherlands in 1953, the Delta Committee asked D. van Dantzig to solve the economic decision problem concerning the optimal height of dikes. This project is considered as the start of Operations Research in the Netherlands. His analysis and formula, published in Econometrica (Van Dantzig (1956)), were still in use in the Netherlands until the first results of the model presented in this paper appeared. Van Dantzig (1956) develops a cost-benefit analysis in which the social costs of investing in water defences are balanced against the social benefit of avoiding damage by flooding. Since non-material issues are involved in social costs and in social benefits alike, the choice of safety standards is ultimately a political decision. We wish to stress that the cost-benefit analysis in this paper is indeed just one of the instruments in the extensive project 'Water safety 21st century' (Kind (2011)), and other criteria will certainly be taken into account for the final decision on the legal flooding standards. The Dutch Water Act defines a standard for the maximum flood probability for all dikes in the Netherlands. These flood probabilities, which are partly based on Van Dantzig's analysis, range from 1/250 per year for small areas in the Meuse valley, via 1/1250 per year for dike rings along the upper part of the Rhine, to 1/10,000 per year for the most important dike rings in the provinces of North and South Holland along the coast. See also Figure 1. These safety standards are probably the highest in the world (see for an overview Galloway et al. (2006), Table 7-1). They are, for instance, much higher than the well-known regular standard in the USA of 1/100 and even much higher than the recently recommended standard of 1/500 for densely populated or vulnerable areas; see Galloway et al. (2006).

Recently, a second Delta Committee argued that the standards stated in the above-mentioned Act are too low in comparison to the present risks, as the standards have not been updated since 1959 (see Deltacommittee, (2008)). Keeping the flood probabilities constant over time is indeed in line with the approach of Van Dantzig, but this seems odd in the light of the growth of both population and wealth. Therefore, the research problem that the government presented to the authors was, first, to develop new flood standards based on the right cost-benefit model. Secondly, the authors were asked to develop solution techniques to solve the resulting optimization model, preferably with a global optimality guarantee. Based on the optimal solutions for the dike rings in the Netherlands, the final goal for the government is to include the new safety standards in an updated version of the Dutch Water Act.

In this paper we show that Van Dantzig's analysis is indeed incorrect with respect to economic growth. According to his approach, the height of a dike after every heightening should be such that the resulting flood probabilities are the same. This strategy is not optimal when considering economic growth, since economic growth implies increasing potential damage by flooding, thus requiring higher dikes to achieve lower flood probabilities. This paper demonstrates how to correctly include economic growth in the cost-benefit analysis. This is our first improvement of Van Dantzig's results. The second improvement concerns the timing of the heightenings. Van Dantzig (1956) only answers the question how much to invest in heightening a dike the first time. He did not really address the 'when' question, for the obvious reason that at that time heightening the dikes was immediately called for. For later heightenings he assumed a priori that they should be heightened periodically, and stated that a reasonable choice for the period T between two heightenings is $T = 1/\eta$, where η is the structural increase of the water level (cm/year). This choice is arbitrary and not necessarily optimal, since it is not a part of the model solution itself. The formulation of the timing problem has been improved by Vrijling and Van Beurden (1990). However, their model does not incorporate economic growth and their solution methods are predominantly numerical. We will see that the solution of our model automatically gives analytical formulas for the optimal heightening intervals and heightening amounts for an even more complicated case in which the investment costs are rising with the existing height of the dike. The formulation chosen for the investment cost function is in accordance with engineering practice, see e.g., Voortman (2002).

We show that for certain choices of the investment cost function there is an explicit periodic solution that satisfies the first order conditions. To cope with other choices of the investment cost function we develop a Dynamic Programming approach. We have calculated the optimal solution for 21 Dutch dike rings, and conclude that several dike rings should indeed be heightened immediately. Moreover, our results suggest that for several dike rings the current safety standards are too low.

In this paper we assume that a dike ring is homogeneous, i.e. that all characteristics for flood probability, investment costs, etc., are the same for all dikes in the dike ring that have to sustain the

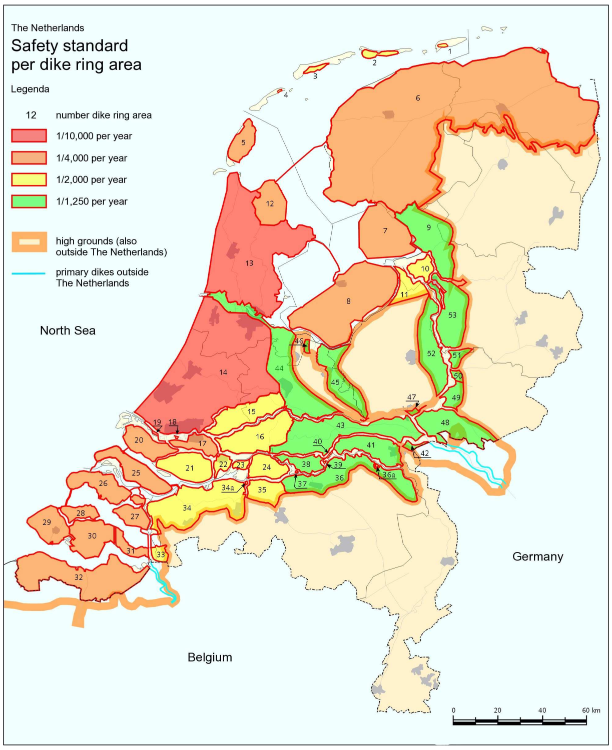

Figure 1: Dike ring areas and safety standards in the Netherlands.

same kind of threat. However, for many dike rings in the Netherlands this is not the case, since a dike ring, especially along rivers, may have different segments, each with different characteristics. The inhomogeneous case is treated in Brekelmans et al. (2012).

This paper is partly based on the reports Eijgenraam (2006), Eijgenraam (2005), and Den Hertog and Roos (2009), and partly on new results.

2 Cost-benefit model

In the first subsection we describe our mathematical model and in the second subsection we comment on the validity of the underlying model assumptions and parameter values.

2.1 Mathematical model

There are 53 so-called dike ring areas in the Netherlands with a higher safety standard than 1/1000 per year; see Figure 1. Each dike ring protects a certain area against flooding. The model that we present is for each separate dike ring or a part of a dike ring with the same kind of threat. In the cost-benefit analysis we attempt to minimize the total social costs, consisting of the investment costs for heightening the dikes and the remaining expected loss by flooding. We model these two cost components.

Expected loss by flooding. The expected annual loss by flooding (S_t) is defined as the product of the potential loss by flooding (V_t) times the probability of a flood per year (P_t). It is assumed that flooding does not occur more than once in the same year. Under normal conditions the dike ring will be properly protected, so the relevant probability distribution is an extreme value distribution for water levels. In practice an exponential distribution fits the data reasonably well (Van Noortwijk et al. (2002)). This distribution is supposed to shift to the right with a constant speed of η centimeters per year, as a result of rising sea levels and higher peak levels of river discharges. The resulting flood probability P_t is the probability that the water level will exceed the level of the dike, resulting in a flooding of the dike ring area. The flood probability P_t is therefore defined in the same way as by Van Dantzig (1956):

$$P_t = P_0^- e^{\alpha\eta t} e^{-\alpha\left(H_t - H_0^-\right)}, \tag{1}$$

where

$$
\begin{aligned}
H_t &= \text{dike height at time } t \text{ (cm)},\\
H_0^- &= \text{dike height just before time } t=0 \text{ (cm)},\\
P_t &= \text{flood probability at time } t \text{ (1/year)},\\
P_0^- &= \text{flood probability just before time } t=0 \text{ (cm)},\\
\alpha &= \text{parameter exponential distribution for extreme water levels (1/cm)},\\
\eta &= \text{structural increase of the water level (cm/year)}.
\end{aligned}
$$

The loss by flooding is defined as:

$$V_t = V_0^- e^{\gamma t} e^{\zeta\left(H_t - H_0^-\right)}, \tag{2}$$

where

$$
\begin{aligned}
V_t &= \text{loss by flooding at time } t \text{ (million euros)},\\
V_0^- &= \text{loss by flooding just before time } t=0 \text{ (million euros)},\\
\gamma &= \text{economic growth rate in dike ring (per year)},\\
\zeta &= \text{increase of loss per cm dike heightening (1/cm)}.
\end{aligned}
$$

The first factor on the right-hand side of (2) also includes a monetary valuation for non-material losses. The second factor on the right-hand side of (2) reflects the economic growth up to year t. We assume a constant growth rate γ. The third factor on the right-hand side of (2) is an addition to Van Dantzig's model and is only relevant along rivers. Along rivers a dike has a slope equal to the slope of the river. Compared to sea level, the tops of the dikes upstream are higher than the tops of the dikes downstream. When flooding occurs, the resulting water level in the dike ring is assumed to always reach as high as the lowest point of the dike above sea level. At this point the water runs over the dike back into the river or into another outlet. Of course, the height of the water level within the dike ring is one of the determinants of the amount of damage. If the dike is heightened, the resulting damage within the dike ring will increase. This is expressed in a simple exponential manner.

Multiplying (1) and (2) leads to the formula for the expected loss at time t

$$S_t = P_t V_t = P_0^- e^{\alpha\eta t} e^{-\alpha\left(H_t - H_0^-\right)} \cdot V_0^- e^{\gamma t} e^{\zeta\left(H_t - H_0^-\right)}, \tag{3}$$

or, equivalently,

$$S_t = S_0^- e^{\beta t} e^{-\theta h_t}, \tag{4}$$

where

$$S_0^- = P_0^- V_0^-, \quad \beta = \alpha\eta + \gamma, \quad \theta = \alpha - \zeta > 0, \quad h_t = H_t - H_0^-. \tag{5}$$

The expected loss S_t increases by β percent per year, as a result of economic growth and of rising water levels due to, among other reasons, climate change. To cope with these systematic changes, defensive actions are required in the future. Here we summarize these defensive actions by 'heightening' of dikes. The corresponding costs are called investment costs.

Throughout this paper only strictly positive values for both parameters β and θ are considered, because other values make no sense in (4). If $\beta < 0$ a possible safety problem in the initial situation would solve itself, because the expected damage would decrease over time. If $\beta = 0$ at most one action solves forever a possible safety problem in the initial situation. Only the case $\beta > 0$ leads to a complicated decision problem with more than one action that has an influence on future actions. In case $\theta \leq 0$, heightening of a dike will be senseless anyway, because then it will not diminish the loss. The restriction to only positive values remains valid if other components would be added to these two composite parameters.

Investment costs. The investment costs for a heightening depend on the amount of the current heightening and the current dike height (i.e., the previous dike heightenings). Van Dantzig (1956) uses a linear investment cost function that does not depend on the height of the current dike. We make the investment costs dependent on the height of the existing dike, since this appears to be more in line with engineering experience. (See also Van Dantzig's remark on this issue on page 280 of Van Dantzig (1956).) Based on engineering practice and detailed cost studies (see e.g. Voortman (2003)) we will consider two specific investment cost functions. The first is an exponential investment cost function

$$I(h^-, u) = \begin{cases} 0 & \text{if } u = 0 \\ (c + bu)\, e^{\lambda(h^- + u)} & \text{if } u > 0, \end{cases} \tag{6}$$

where c, b and λ are positive constants, and h^- is the height of the dike just before a heightening by u.

The second is a quadratic investment cost function

$$I(h^-, u) = \begin{cases} 0 & \text{if } u = 0 \\ a_0 \left(h^- + u\right)^2 + b_0 u + c_0 & \text{if } u > 0, \end{cases} \tag{7}$$

for suitably chosen constants a_0, b_0 and c_0. Note that both investment cost functions are not continuous at $u = 0$, due to the fixed costs. This is the reason that the dike heightening process is discontinuous and non-convex.

Overall model. Social welfare is maximized by minimizing the present value of the total expected costs of flooding and investment over the whole future. The dike height is a nondecreasing step function that is increased at moments $t_1 < t_2 < \ldots$ with a certain amount in order to increase the safety level. We denote the increment of the height at time t_k as u_k. We also introduce the notation $t_0 = 0$. Note that it may happen that $t_1 = t_0 = 0$, in which case there is an immediate increase. Using this notation we have $h_t = H_t - H_0^- = \sum_{i=1}^k u_i$ for $t_k \le t < t_{k+1}$. Note that $h_{t_k} = \sum_{i=1}^k u_i$. We use this notation to rewrite the expected total discounted damage costs as

$$\int_0^\infty S_t e^{-\delta_1 t} dt = \sum_{i=0}^\infty \int_{t_i}^{t_{i+1}} S_0^- e^{\beta t} e^{-\theta h_{t_i}} e^{-\delta_1 t} dt = \frac{S_0^-}{\beta - \delta_1} \sum_{i=0}^\infty e^{-\theta h_{t_i}} [e^{(\beta - \delta_1) t_{i+1}} - e^{(\beta - \delta_1) t_i}] \tag{8}$$

and the total investment costs as

$$\sum_{k=1}^\infty I\left(h_{t_{k-1}}, u_k\right) e^{-\delta t_k},$$

where we use different discount rates δ_1 and δ for the damage and investment costs that have the following relationship:

$$\delta_1 \quad = \quad \delta + \rho \text{ (1/year)}.$$

Hence the optimization problem can be formulated as follows:

$$\min \left\{ \frac{S_0^-}{\beta - \delta_1} \sum_{i=0}^\infty e^{-\theta \sum_{l=1}^i u_l} [e^{(\beta - \delta_1) t_{i+1}} - e^{(\beta - \delta_1) t_i}] + \sum_{i=1}^\infty I\left(\sum_{l=1}^{i-1} u_l, u_i\right) e^{-\delta t_i} \right\}, \tag{9}$$

in which the optimization variables are $u_1, u_2, \ldots$ and $t_1, t_2, \ldots$.

2.2 Comments on model assumptions and parameter values

The model described above relies on several crucial assumptions, and the question is whether these are justified. Moreover, the model contains several important parameters. We argue that the parameter values used in this paper are reasonable, however for the final decision several separate more detailed studies have been carried out to determine the parameter values more accurately taking into account more hydraulic and local aspects (see Kind (2011)). Van Dantzig (1956) already extensively discussed several assumptions and parameter values in his model. In this section we motivate several choices made in our model.

Investment costs. For an accurate estimation of the investment costs we used the results of numerous extensive Dutch studies on this issue, which were combined and summarized by Sprong (2008). Using these cost estimation results, the parameters for the exponential (6) and quadratic investment cost functions (7) were fitted for each dike ring. Note that in both cost functions, the costs of heightening the dike depend on the current height. This is obvious, since the dikes also need to be made wider, which is more costly for higher dikes.

As also extensively described in Opperman et al. (2009), the sinking of the deltas due to human activity is also a major problem. However, it makes no difference for our mathematical model if we should implement measures that decrease water levels instead. Since the latter type of measures is generally not possible along the coast, we will continue to focus only on heightening dikes. Also note that heightening dikes is generally cheaper than lowering design-water levels by giving a river more space, e.g., by enlarging the distance between the dikes along a river.

To place more emphasis on the current heightening by u, one could use the following alternative form for the exponential investment costs (6):

$$I(h^-, u) = (c + bu)\, e^{\lambda_1 h^- + \lambda_2 u}, \quad u > 0, \tag{10}$$

where λ_1 and λ_2 are different positive constants, with $\lambda_2 > \lambda_1$. In this paper we assume $\lambda_1 = \lambda_2 = \lambda$. It may be worth mentioning, however, that the Dynamic Programming approach proposed in Section 4 still works for cases where λ_1 and λ_2 are different.

It seems natural to require that there is no advantage to perform a heightening in two parts. For example, if a dike heightening of 100 cm is required, it should be cheaper to do this all at once, rather than first heightening the dike by 30 cm, and subsequently by 70 cm. Hence, a natural requirement seems to be that

$$I(h^-, u_1 + u_2) \leq I(h^-, u_1) + I(h^- + u_1, u_2), \quad \forall u_1, u_2 > 0. \tag{11}$$

It can easily be checked that the quadratic investment cost function (7) satisfies this condition. The exponential investment cost function satisfies the above condition when we have

$$bu_1 \left(e^{\lambda u_2} - 1\right) \leq c, \quad \forall u_1, u_2 > 0. \tag{12}$$

We have verified that indeed this condition is satisfied for all dike rings.

The cost functions for heightening dikes do not distinguish between the first and next dike heightenings. This distinction may be necessary in several practical situations. It is easy to see that the Dynamic Programming approach in Section 4 still works for such situations.

Damage costs. The value of the damage costs V_0^- for a certain dike ring is based on extensive simulations by engineers of Deltares, using the information system HIS-SSM. Cost categories such as for evacuation and rescue and immaterial damage costs (e.g. victims, suffering) were added, since they were not included in this information system.

The value for the growth rate γ is based on growth scenarios for the Netherlands made by CPB Netherlands Bureau for Economic Policy Analysis. This growth rate is used for the material as well as immaterial damage. We therefore assume an income elasticity of one for some parts of the immaterial damage.

In the model, the damage by a flood along the rivers depends on the height of the dike at that moment. This is obvious, since the water level in such a dike ring area after a flood will be high if the dikes are high. Research by engineers of Rijkswaterstaat (the implementing body of the Ministry of Infrastructure and Environment) has shown that this can be expressed in a simple exponential manner. This is a reasonable approximation for the change in the amount of loss within a relevant range for the heightening of the dike along the rivers; see Kind (2011).

As pointed out in many papers, it may be dangerous to focus only on expected values when low probabilities are combined with high costs. This is certainly a danger in our case, since the flood probabilities are small and the damage costs are high. Van Dantzig (1956) already considered the question whether a cost-benefit approach is appropriate: "... but also because probability principles are here used in a case where there may not seem to be an adequately large number of comparable social risks to make the concept of mathematical expectation a suitable basis for social choice". The analysis of Stern (2006) is also based on a cost-benefit analysis, and several economists have criticized that. For example, Weitzman (2007) argues that Stern places too much emphasis on a cost-benefit analysis and too little on the need for social insurance against low-probability catastrophic events. One may therefore argue that a so-called risk-premium should be included. This can be done via the discount rate by choosing $\delta_1 < \delta$ or more explicitly in the valuation of the damage itself. We show that our model allows for such an addition, although it was finally decided to include this last type of risk-premium in the actual calculations in another way.

To obtain a robust formulation, it is often advised to take higher-order moments into account. In the field of asset liability, the mean-variance model, introduced by Markowitz (1952), is often used. Vrijling et al. (1998) also propose to add second-order moments in probabilistic design models, including the design of dikes.

First note that since the investment costs are deterministic, the variance of these costs is zero. Hence, it suffices to concentrate on the variance of the costs of flooding. We first derive expressions for the second-order moments, and then show how to incorporate these moments into the mathematical optimization models, which is the approach advocated by Vrijling et al. (1998).

The random variable Z_t denotes the actual costs of flooding in year t. In year t we have the following probabilistic situation: flooding occurs with probability P_t and no flooding with probability $1 - P_t$. It can easily be verified that the first two moments of Z_t are:

$$E(Z_t) = P_t V_t = S_t,$$

and

$$\mathrm{Var}(Z_t) = P_t(1 - P_t)V_t^2 \approx P_t V_t^2,$$

where the last approximation follows because P_t is very small. Using the results and notation of (4) and (5), for the second-order moments we obtain

$$\mathrm{Var}(Z_t) \approx P_t V_t^2 = S_0' e^{\beta' t} e^{-\theta'(H_t - H_0^-)}, \tag{13}$$

where

$$S_0' = P_0^-(V_0^-)^2, \quad \beta' = \alpha\eta + 2\gamma, \quad \theta' = \alpha - 2\zeta, \tag{14}$$

and

$$\sigma(Z_t) = \sqrt{\mathrm{Var}(Z_t)} \approx \sqrt{P_t}\, V_t = S_0^* e^{\beta^* t} e^{-\theta^*(H_t - H_0^-)}, \tag{15}$$

where

$$S_0^* = \sqrt{P_0^-} V_0^-, \quad \beta^* = \frac{1}{2}\alpha\eta + \gamma, \quad \theta^* = \frac{1}{2}\alpha - \zeta. \tag{16}$$

This means that both $\mathrm{Var}(Z_t)$ and $\sigma(Z_t)$ have the same structure as S_t, only with slightly different values for the constants S_0^-, β, and θ. However, one should remain careful as it may happen that θ' or θ^*, contrary to θ, becomes negative, which would mean that heighthening a dike will lead to more damage instead of diminishing the expected loss. Finally, it may be better to use $\sigma(Z_t)$ instead of $\mathrm{Var}(Z_t)$ for scalability reasons, since $\sigma(Z_t)$ has the same dimension as $E(Z_t)$. Note, however, that $\sigma(Z_t)$ is still much greater than $E(Z_t)$. See also Vrijling et al. (1998) for possible solutions to this scalability problem. A possible way to include this risk measure in our model is to add the standard deviation multiplied by a positive scalar κ to the objective. This does not fit in the standard formulation (9), but can easily be added in the Dynamic Programming approach.

Flood probability and sea level rise. Van Dantzig (1956) also uses an exponential distribution for the flood probabilities. He indicates that this assumption may be questionable. For example, Dillingh et al. (1993) uses extreme value theory to arrive at predictions for extreme water levels. However, in our model the flood probability not only accounts for extreme water levels but also for the distribution of the wind directions and forces, as well as for various local circumstances. Van Noortwijk et al. (2002) show that an exponential distribution indeed yields a good approximation for the flood probabilities. We therefore decided to keep the exponential distribution, but wish to point out that the Dynamic Programming approach in Section 4 still works when using other distributions, such as the Weibull distribution.

In our model we assume that, for instance due to the relative sea level rise, the distribution will shift though the form of the distribution remains unchanged. This is possible because the distribution function is only valid for very high water levels that have a probability of occurrence of less than 0.2 percent per year. If more knowledge on this issue becomes available, then perhaps the Weibull distribution can provide more degrees of freedom to model this.

An implicit assumption in our model is that a flooding in a certain dike ring does not influence the probability of flooding in other dike rings. This assumption has been validated by Deltares. Moreover, in our model we assume that if a flood occurs, the dikes are repaired immediately. In reality this will take approximately until the next storm season, so the effect of this is negligible. Moreover, we assume that dike heightenings are measured when they are completed, thereby decreasing the flood probability immediately.

Much research has been done concerning the sea level rise in the future. Often such research is based on Intergovernmental Panel on Climate Change (IPCC) global climate change scenarios. In our study we use the sea level scenarios of the Royal Netherlands Meteorological Institute (KNMI), see Katsman et al. (2008), which are also based on the IPCC scenarios.

Discounting. Especially after the famous Stern Review (Stern (2006)), a major debate has been ongoing about the correct discount rate in the cost-benefit analysis of climate economics. Stern (2006) uses 1.4% for the discount rate (including a 0.1% risk premium), which is much lower than the $5-6\%$ used in many other climate economics models. It is clear that a higher discount rate leads to a postponement of investments in dike heightening. Ackerman (2009) concludes that in the end, the choice of the discount rate is an ethical and political choice.

In March 2007, the Dutch government fixed the value for the real discount rate in a cost-benefit analysis. At that time the rate consisted of two components: the real risk-free discount rate, the value of which the government fixed to 2.5% per year, and a macro-economic risk premium, which value has to be estimated for each project separately. If one does not investigate the risk profile of the project, one has to use 3%-points for the macro-economic risk premium. In September 2009 the Dutch government decided that, for specific types of effects, the risk premium should be halved to 1.5%-points. This negative third component represents the lowering of risk by projects that so to speak have some insurance character. The project has to prevent negative external effects of an irreversible nature that carry a monetary value in the cost-benefit analysis. Since a large part of the loss by flooding is irreversible, such as loss of human lives, anxiety and hardship during and after the flooding and loss of personal belongings, dikes are a good example of a project with effects that have to be discounted with half a risk premium. See Aalbers (2009) for the economic background study on this issue. However, in the final cost-benefit analysis not this method via different discount rates but a direct mark-up on (parts of) the damage costs has actually been used to deal with risk-aversion.

In the calculations presented in this paper, the risk premium has also been differentiated between the two components in the objective function. Based on a historical analysis, it is assumed that there is no correlation between the actual development of the price index of investment costs and the actual development of the volume index of the economy in the Netherlands. In that case, δ is the real risk-free discount rate. The parameter ρ is an estimate of the macro-economic risk premium, diminished by the insurance character of the project. Therefore, a risk premium of 1.5% is used, so δ_1 will be set at 4% per year. However, this system of discounting different items in different ways is now under discussion. Using one or two discount rates does not change the structure of the model. To simplify notation, we redefine $\beta = \alpha\eta + \gamma - \rho$ (cf. (5)), and use $\delta_1 = \delta$ in the remainder of this paper.

There is also much discussion among economists whether the discount rate should be kept constant over time or declining. Several theories have been developed that conclude that the discount rate should start out high, and decline in the future. However, as observed by Ackerman (2009), this has only a limited impact in practice: "A high discount rate for the first few decades accomplishes most of the shrinkage of future values; after that, it does not much matter whether the rate goes down".

In the model we apply continuous compounding, and one may ask why discrete compounding was not applied. In Den Hertog and Roos (2009) it is argued that the difference between continuous and discrete compounding is not very significant. Since continuous compounding leads to a much simpler analysis (see (8)) than discrete compounding, we have chosen for continuous compounding.

Finally, we emphasize our assumption that the parameter values are constant over time.

3 Periodic solutions

In this section we show that if the exponential investment cost function (6) is used, there is a periodic solution that satisfies the first order conditions. This periodic solution can be derived analytically. We label a solution *periodic from heightening l on*, when for some $\nu > 0$ and $p > 0$ it holds that

$$u_j = \nu \quad \text{for all} \quad j \geq l, \tag{17}$$

and

$$t_j = t_l + (j-l)p, \quad j \geq l. \tag{18}$$

We call a solution *periodic* if there exists an $l \geq 1$ such that this solution is periodic from heightening l on. In addition, we analyze which other investment cost functions may lead to periodic stationary solutions. The quadratic investment cost function (7) is not among these investment cost functions.

3.1 Exponential investment cost function

3.1.1 Theoretical analysis

In this subsection we consider the exponential investment cost function (6). We first derive the first-order conditions for a stationary point of the objective in (9). To do so, we calculate all partial derivatives with respect to u_i and t_i and set them equal to zero. For the derivatives with respect to t_i this leads to the well known criterion that at the moment of investment the First Year Rate of Return (FYRR) has to be zero. Then we prove that the first order conditions have a periodic solution as defined by (17) and (18) from heightening $l = 2$ on. This somewhat tedious task is performed in Den Hertog and Roos (2009). The final results are

$$p = \frac{\theta + \lambda}{\beta} \nu, \tag{19}$$

and ν is the solution of the following equation:

$$\frac{b}{(c + b\nu)} \left(e^{\theta\nu} - 1\right) + \left[\lambda \left(e^{\theta\nu} - 1\right) - \frac{\theta\delta}{\beta - \delta} \left(e^{(\theta - q)\nu} - 1\right)\right] \frac{1}{1 - e^{-q\nu}} = 0, \tag{20}$$

with

$$q = \frac{\delta\theta - (\beta - \delta)\lambda}{\beta}, \tag{21}$$

and δ has to be large enough such that $q > 0$. In Den Hertog and Roos (2009) we prove that this equation always has an unique positive solution, which can easily be computed using binary search.

For t_1 we need to distinguish two cases depending on the size of the expected damage S_0^- at the start compared to the optimal size of the expected damage just before the first normal investment after the start. Therefore, it is handy to define $s(\nu)$:

$$s(\nu) = \frac{\delta (c + b\nu) e^{\lambda\nu}}{1 - e^{-\theta\nu}}.$$

Case I: without back maintenance.

This case, in which back maintenance is absent, i.e., $S_0^- \leq s(\nu)$, meaning that there is in general no need to heighten the dike immediately, is identified by the fact that t_1 as defined below is positive. In Den Hertog and Roos (2009) we prove that in Case I the stationary point is the periodic solution specified above in (19) and (20) from heightening $l = 1$ on and:

$$t_1 = \frac{1}{\beta} \ln \frac{s(\nu)}{S_0^-}. \tag{22}$$

At time t_1 the size of the expected damage has increased to $s(\nu)$, the FYRR criterion is met and the first periodic heightening takes place.

Case II: with back maintenance.

This case is identified by the fact that t_1 in (22) is negative. If this is the case, then we should already have heightened the dike ring in the past, i.e., we have a backlog because $S_0^- > s$. Hence in this case we need a heightening at $t_1 = t_0 = 0$. InDen Hertog and Roos (2009) we prove that in this case a stationary point has the periodic solution specified above in (19) and (20) from heightening $l = 2$ on and:

$$t_1 = 0,$$

$$t_2 = \frac{1}{\beta} \left[(\theta + \lambda) u_1 + \ln \frac{s(\nu)}{S_0^-}\right] > 0.$$

Moreover, u_1 can be found by minimizing the univariate function

$$(c + bu_1) e^{\lambda u_1} - \frac{S_0^-}{\beta - \delta} e^{-\theta u_1} + g(\nu) e^{-q u_1},$$

where

$$g(\nu) = \frac{\beta (c + b\nu) e^{\lambda\nu}}{(\beta - \delta)(1 - e^{-q\nu})} \left(\frac{S_0^-}{s(\nu)}\right)^{\frac{\delta}{\beta}},$$

under the conditions that $t_2 > 0$ and

$$\frac{q+2\lambda}{q+\lambda}b + \lambda(c + bu_1) - \frac{\theta}{\delta}S_0^- e^{-(\theta+\lambda)u_1} > 0.$$

The last condition assures that the solution found is a minimum. In Den Hertog and Roos (2009) it is shown that this univariate minimization can easily be done.

The solution given by either Case I or Case II is a stationary point. To prove that this solution is the global optimum, we have to show that this periodic solution also satisfies the second-order sufficient conditions. One can try to use the impulse-maximum principle (see Feichtinger and Hartl (1986)) to prove optimality for the periodic solution. Although our problem (9) can easily be seen as an impulse control problem, the second-order conditions for impulse control problems are not fulfilled, since the investment cost function has fixed costs and therefore is not convex in the 'control variable' u_i. Numerical experiments in Section 4, however, show that this periodic solution is indeed the global optimum. More numerical evidence for the global optimality of the periodic solution can be found in Den Hertog and Roos (2009).

3.1.2 Numerical results

No.	c	b	λ	α	η	ζ	V_0^-	P_0^-
10	16.6939	0.6258	0.0014	0.033027	0.320	0.003774	1564.9	0.00044
11	42.6200	1.7068	0.0000	0.032000	0.320	0.003469	1700.1	0.00117
15	125.6422	1.1268	0.0098	0.050200	0.760	0.003764	11810.4	0.00137
16	324.6287	2.1304	0.0100	0.057400	0.760	0.002032	22656.5	0.00110
22	154.4388	0.9325	0.0066	0.070000	0.620	0.002893	9641.1	0.00055
23	26.4653	0.5250	0.0034	0.053400	0.800	0.002031	61.6	0.00137
24	71.6923	1.0750	0.0059	0.043900	1.060	0.003733	2706.4	0.00188
35	49.7384	0.6888	0.0088	0.036000	1.060	0.004105	4534.7	0.00196
38	24.3404	0.7000	0.0040	0.025321	0.412	0.004153	3062.6	0.00171
41	58.8110	0.9250	0.0033	0.025321	0.422	0.002749	10013.1	0.00171
42	21.8254	0.4625	0.0019	0.026194	0.442	0.001241	1090.8	0.00171
43	340.5081	4.2975	0.0043	0.025321	0.448	0.002043	19767.6	0.00171
44	24.0977	0.7300	0.0054	0.031651	0.316	0.003485	37596.3	0.00033
45	3.4375	0.1375	0.0069	0.033027	0.320	0.002397	10421.2	0.00016
47	8.7813	0.3513	0.0026	0.029000	0.358	0.003257	1369.0	0.00171
48	35.6250	1.4250	0.0063	0.023019	0.496	0.003076	7046.4	0.00171
49	20.0000	0.8000	0.0046	0.034529	0.304	0.003744	823.3	0.00171
50	8.1250	0.3250	0.0000	0.033027	0.320	0.004033	2118.5	0.00171
51	15.0000	0.6000	0.0071	0.036173	0.294	0.004315	570.4	0.00171
52	49.2200	1.6075	0.0047	0.036173	0.304	0.001716	4025.6	0.00171
53	69.4565	1.1625	0.0028	0.031651	0.336	0.002700	9819.5	0.00171

Table 1: Input data for 21 dike rings.

From Rijkswaterstaat/Deltares we received data for all dike rings in the Netherlands. These data were generated by water experts, sometimes using extensive simulations. The parameter values for 21 dike rings along branches of the river Rhine are given in Table 1. We computed solutions for two combinations of the discount rate δ and ρ, namely $\delta = 0.025$ and $\rho \in \{0.015, 0.025\}$. These are shown in Table 2. Moreover, in Figure 2 the flood probabilities for the optimal solutions for two dike rings are given. From the results we make the following observations.

From Figure 2 we observe that the flood probability increases within an interval between two heightenings, because of the expected rise of river discharges. Also note that if we take the probabilities just after a heightening into account, then these probabilities decrease over time. This is due to the fact that we take economic growth into account, which leads to increasing damage costs and thus requires lower flood probabilities.

	$\rho = 0.015$, $\delta = 0.025$					$\rho = 0.025$, $\delta = 0.025$				
No.	year t_1	u_1	p	ν	costs	year t_1	u_1	p	ν	costs
10	31	54	105	54	67.4	75	41	225	41	52.4
11	24	58	109	58	181.1	58	43	236	43	146.3
15	0	78	73	56	820.2	0	75	88	52	701.2
16	0	71	76	56	1634.5	0	68	88	52	1418.0
22	0	64	88	58	450.0	0	60	102	53	407.3
23	46	59	68	59	51.0	57	55	79	55	40.3
24	0	81	59	66	480.4	0	78	69	62	419.8
35	0	91	59	62	575.5	0	89	71	58	475.6
38	0	93	96	59	221.2	0	84	211	46	185.3
41	0	129	116	71	395.5	0	117	251	55	342.6
42	0	84	111	69	103.7	0	72	220	54	89.2
43	0	91	118	70	1654.9	0	79	241	55	1400.5
44	0	98	104	47	256.8	0	91	242	36	220.1
45	0	81	95	39	43.1	0	76	207	31	36.5
47	0	91	96	52	79.6	0	82	211	40	68.6
48	0	84	78	49	550.7	0	79	160	39	449.3
49	3	43	98	43	114.9	0	34	215	33	93.9
50	0	124	108	58	61.4	0	111	227	44	55.6
51	11	38	96	38	87.3	18	30	208	30	69.7
52	0	62	106	43	308.0	0	54	222	34	262.0
53	0	110	126	62	361.9	0	99	274	49	319.2

Table 2: Results for the 21 dike rings in Table 1.

It emerges that, for almost all dike ring areas, the current safety levels are lower than calculated via our model and therefore need an immediate heightening. Dike ring area 15 is one of these, since there is an immediate heightening at $t_1 = 0$. See e.g. Figure 2. It appears that 16 out of the 21 dike rings studied in this paper need an immediate heightening. The reason for this is that the estimates for P_0^- were raised considerably as a consequence of the very high river discharges in both 1993 and 1995. The Dutch project "Room for the River", started in 2006, aims to bring the safety levels back to their legal values by 2015.

The figures in Table 2 show that for some dike rings the length of the period p and the number of years till the first heightening (t_1) are quite sensitive to the risk premium (and thus to the economic growth rate and the velocity of relative sea level rise). For example, for dike ring 11 the values of t_1 are 24 and 58, respectively.

3.2 Other investment cost functions

In this subsection we discuss which classes of investment cost functions can be met with a periodic solution that satisfies the necessary first-order conditions.

Using the analysis in Den Hertog and Roos (2009) it can be shown that such investment cost functions should be such that

$$\frac{I\left(h_{t_j}, u_{j+1}\right)}{I(h_{t_{j-1}}, u_j)} = \frac{I\left(u_1 + (j-1)\nu, \nu\right)}{I(u_1 + (j-2)\nu, \nu)} \tag{23}$$

is independent of j, for $j \geq 2$. We proceed by checking if this condition is satisfied for several investment cost functions. We start with the two investment cost functions considered before.

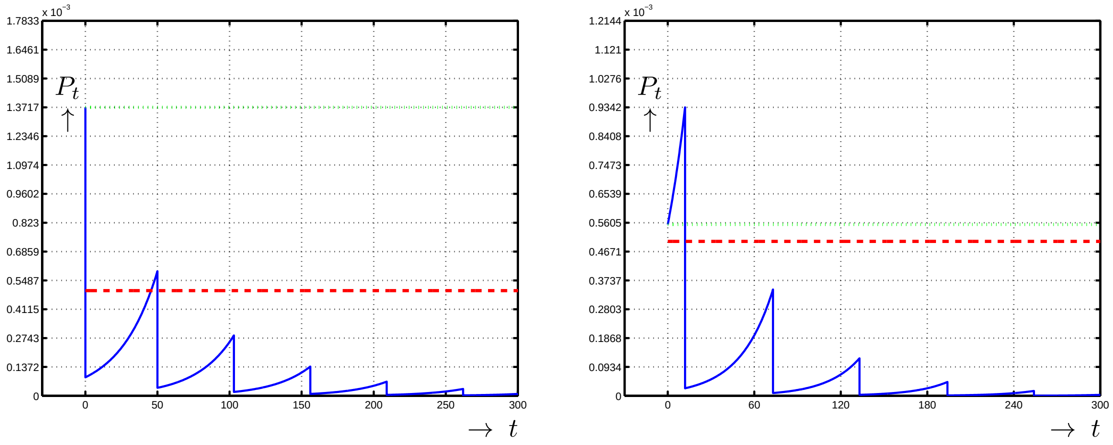

Figure 2: Solution for dike ring 15 (left) and 22 (right) with exponential investment costs. The blue curves represent P_t, the green lines P_0^-, and the red lines the current safety standard.

For the exponential investment cost function (6) we have

$$\frac{I\left(u_1+(j-1)\nu,\nu\right)}{I(u_1+(j-2)\nu,\nu)}=\frac{(c+b\nu)e^{\lambda(u_1+j\nu)}}{(c+b\nu)e^{\lambda(u_1+(j-1)\nu)}}=e^{\lambda\nu},$$

which is independent of j. This is in agreement with the results in Section 3.1, namely that for the exponential investment cost function there is a periodic solution that satisfies the first order conditions.

For the quadratic investment cost function (7), we have

$$\frac{I\left(u_1+(j-1)\nu,\nu\right)}{I(u_1+(j-2)\nu,\nu)}=\frac{a_0\left(u_1+j\nu\right)^2+b_0\nu+c_0}{a_0\left(u_1+(j-1)\nu\right)^2+b_0\nu+c_0};$$

since $a_0>0$, the last expression is dependent on j. Hence, the quadratic investment cost function will not yield an optimal solution that is periodic. This is in accordance with the numerical results found in Section 4.2.

We discussed an exponential investment cost function (10) that can be used to give more (or less) emphasis to the last heightening than to the previous heightenings. For this function we have

$$\frac{I\left(u_1+(j-1)\nu,\nu\right)}{I(u_1+(j-2)\nu,\nu)}=\frac{(c+b\nu)e^{\lambda_1(u_1+(j-1)\nu)+\lambda_2\nu}}{(c+b\nu)e^{\lambda_1(u_1+(j-2)\nu)+\lambda_2\nu}}=e^{\lambda_1\nu},$$

which is independent of j. This implies that (10) satisfies (23).

A final observation is that if we have two investment cost functions $I(h,u)$ and $\tilde{I}(h,u)$ that satisfy the above condition for a periodic stationary point, then so does their product $I(h,u)\times\tilde{I}(h,u)$.

4 Dynamic Programming approach

4.1 The method

In the previous section we showed that for certain classes of the investment cost function, we can find the solution analytically. However, for quadratic investment cost functions (7), problem (9) cannot be solved analytically. In this section we show that this problem can be solved using Dynamic Programming (DP). This approach can also be used if, for example, other choices for the damage costs or the flood probability are made.

In the Dynamic Programming approach we distinguish *stages* and *states* of the process under consideration, and *transitions* from one state in a certain stage to another state in the next stage. Each transition is associated with costs, and the aim is to find a sequence of transitions starting at the initial state and ending at a desired state that minimizes the total costs of these transitions. Finding such a sequence can be achieved efficiently by using a recursive relation.

First of all we truncate the infinite horizon in (9) to T years. Let us define the stages as the years $t = -1, 0, 1, 2, ..., T$, in which $t = -1$ is the time just before $t = 0$. Let us define a state at stage t as h_t, where h_t is a possible height of the dike at time t. The initial state at stage $t = -1$ is $h_{-1} = 0$, the current height of the dike. We use a 'safe' upper bound $\bar{H}$ for a safe dike height at the end of the planning horizon. We discretize the possible height (we will use cms). It is very easy to give a priori bounds for the flood probabilities. We use these upper bounds to restrict the possible states:

$$P_t = P_0^- e^{\alpha\eta t} e^{-\alpha\left(H_t - H_0^-\right)} \leq P_{\max}, \quad \forall t.$$

By taking logarithms at both sides, while using the definition of h_t in (5), we obtain

$$h_t \geq \eta t + \frac{1}{\eta} \ln \frac{P_0^-}{P_{max}}.$$

Now let $\mathcal{H}_t$ denote the set of all feasible dike heights at time t.

We assume that a dike heightening is accomplished within one time unit. Then, in our model a transition can occur from state h_t in stage t to state h_{t+1} in stage $t + 1$ only if $h_{t+1} \geq h_t$. We denote the corresponding transition costs as $c_t(h_t, h_{t+1})$. These costs consist of the investment costs (which are positive only if $h_{t+1} > h_t$), and the expected damage costs in the period $[t, t + 1]$, for $t = 0, 1, ..., T - 1$:

$$c_t(h_t, h_{t+1}) = \int_t^{t+1} S_t e^{-\delta t} dt + I(h_t, h_{t+1} - h_t)\, e^{-\delta(t+1)}.$$

For $t = -1$ we have

$$c_{-1}(h_{-1}, h_0) = I(0, h_0).$$

The Dynamic Programming approach is based on the recursive relation

$$f_t(h_t) = \min_{h_t \leq h_{t+1} \in \mathcal{H}_{t+1}} \left\{ f_{t+1}(h_{t+1}) + c_t(h_t, h_{t+1}) \right\}, \quad t < T, h_t \in \mathcal{H}_t, \tag{24}$$

in which $f_t(h_t)$ denotes the minimal costs to cover years $t, t + 1, ..., T, T + 1, ..., \infty$, starting in state h_t.

Now let's take a careful look at $f_T(h)$, since we truncate the infinite horizon at $t = T$. Not taking into account what happens after T, minimizing total costs before $t = T$ will result in postponing investments, so that after $t = T$, high investments may have to be made. To avoid this, and to make our models more realistic at the end of the planning period, we take into account costs after the planning horizon. To that end, it is common in Dynamic Programming approaches to add a so-called salvage term, which can be done in several ways. However, we assume that after the planning horizon there are no changes in the system (i.e., $\beta = 0$ and hence $\rho = 0$), and there are no dike heightenings and hence no investment costs after the planning horizon. The expected damage after T is then given by

$$S_T \int_T^\infty e^{-\delta t} dt = \left. \frac{S_T e^{-\delta t}}{-\delta} \right|_T^\infty = \frac{S_T e^{-\delta T}}{\delta}.$$

Hence, we have

$$f_T(h) = \frac{S_T e^{-\delta T}}{\delta}. \tag{25}$$

Using this formula in the recursion formula (24), we can compute the optimal solution, starting at the last stage T.

Note that this Dynamic Programming approach is very flexible with respect to, for instance, the choice for the investment cost function or the flood probability. As long as we can (numerically) compute the transition costs we can apply the Dynamic Programming approach. Other expressions for, say, P_t or V_t, can be used as well, and there is no need for functions to be convex, for example.

4.2 Numerical results

In Table 3 we summarize the input data for the quadratic investment cost function (7) for 5 dike rings. We calculated the Dynamic Programming solutions for these dike rings using both the exponential (6) and the quadratic investment cost functions (7). In our experiments the salvage term (25) was included in the objective function. We took the number of height levels as 50 and $T = 300$. No probability constraints were used. The running time varied between 3.9 and 4.5 seconds on a Microsoft Windows PC (1.86 GHz, 3.5 GB RAM) under Matlab. We used $\delta = 0.04$ and $\rho = 0$ in the calculations. The periodic solutions for the case of exponential investment costs are given in Table 4.

No.	10	11	15	16	22
a_0	0.0004	0	0.027	0.102	0.0154
b_0	0.7637	1.7168	3.779	3.1956	2.199
c_0	12.603	42.003	67.699	319.25	141.01

Table 3: Data for the quadratic investment cost function for dike rings 10, 11, 15, 16, 22.

No.	ν	p	u_1	t_1	t_2	Inv.	Dam.	Tot. costs
10	56.96	57.12	56.96	45.80	102.92	10.20	29.84	40.04
11	62.42	58.89	62.42	42.44	101.33	30.18	80.05	110.23
15	53.29	51.54	55.96	0.00	51.20	415.50	129.68	545.18
16	52.59	54.04	52.59	3.50	57.54	797.65	292.03	1089.68
22	53.70	62.43	53.70	12.72	75.16	198.53	110.72	309.25

Table 4: Periodic solutions for dike rings 10, 11, 15, 16, 22, with $\delta = 0.04$, $\rho = 0$, and exponential investment costs.

The Dynamic Programming results for the exponential and quadratic investment cost functions are presented in Tables 5 and 6, respectively. From the results we draw the following observations.

Comparing the Dynamic Programming solutions for the exponential investment cost function in Table 5 with those in Table 4, one can observe that the (total) cost values are almost the same. There are three obvious sources for the (minor) differences: first, the truncation to a finite horizon T; second, the discretization of the problem in the Dynamic Programming approach, which inherently introduces some inaccuracy; and third, numerical inaccuracy in the computations. Given this, it is surprising that the differences are so small.

Moreover, the Dynamic Programming solutions for the exponential investment cost function are almost periodic. Only at the end of the planning period there seems to be no periodicity. But this is in all likelihood due to the truncation of the planning period, which disturbs the periodicity at the end of the planning period. This was confirmed when we ran our program with $T = 600$ instead of $T = 300$. In all cases we obtained solutions that were periodic with a few exceptions at the end of the planning period. This is in accordance with the analysis given in Section 3.1. The solution for the quadratic investment cost function is clearly not periodic, which is in accordance with the analysis given in Section 3.2.

The difference in the results between the exponential and quadratic investment cost functions is not very significant. It seems that the quadratic investment cost function tends to perform the first heightenings slightly earlier, but the amount of heightening is lower. The time between two heightenings seems to increase over time for the quadratic investment cost function, while for the exponential investment cost function the time between two heightenings is more or less constant over time.

There are several dikes that need to be heightened immediately. Dike ring area 15 is one of them, since there is an immediate heightening at $t_1 = 0$. Note that this is the case for both the exponential and the quadratic investment cost functions.

No.	10	11	15	16	22
Heightenings	46 : 57.60	43 : 63.36	0 : 54.72	4 : 54.72	12 : 52.08
$(t_k : u_k)$	104 : 57.60	103 : 63.36	50 : 54.72	60 : 54.72	73 : 52.08
	162 : 57.60	162 : 61.44	103 : 54.72	116 : 54.72	133 : 52.08
	219 : 55.68	220 : 61.44	156 : 54.72	171 : 50.16	194 : 52.08
	274 : 51.84	277 : 53.76	209 : 54.72	223 : 50.16	254 : 52.08
			262 : 54.72	274 : 45.60	
h_T	280.32	303.36	328.32	310.08	260.4
Investment costs	10.16	29.70	413.39	796.31	202.09
Damage costs	29.87	80.54	131.95	294.13	107.33
Total costs	40.04	110.23	545.34	1090.44	309.41

Table 5: Dynamic Programming solutions for 5 dike rings, with exponential investment costs.

No.	10	11	15	16	22
Heightenings	46 : 53.76	43 : 61.44	0 : 45.60	3 : 50.16	12 : 48.36
$(t_k : u_k)$	99 : 53.76	101 : 63.36	42 : 50.16	59 : 54.72	69 : 52.08
	155 : 57.60	160 : 61.44	92 : 59.28	118 : 63.84	131 : 55.80
	214 : 61.44	218 : 59.52	149 : 68.40	183 : 68.40	197 : 59.52
	277 : 55.68	272 : 42.24	212 : 77.52	250 : 72.96	265 : 55.80
			280 : 63.84		
h_T	282.24	288.00	364.80	310.08	271.56
Investment costs	9.97	29.33	418.94	840.70	205.15
Damage costs	30.17	80.90	163.35	317.51	112.09
Total costs	40.14	110.24	582.28	1158.21	317.24

Table 6: Dynamic Programming solutions for 5 dike rings, with quadratic investment costs.

5 Concluding remarks

5.1 Contribution to OR and public interest

In the paper we defined an important policy problem: to determine the safety standards for the dikes in the Netherlands to protect against floods, using a cost-benefit analysis. Moreover, we developed a novel model for this problem, and developed a method to solve the model. The model and method were implemented as a user-friendly software package by the company HKV. This software has by now been used to provide the Dutch government with specific policy recommendations with respect to dike safety standards (see Kind (2011)). It is expected that in the near future the Dutch Water Act will be adapted accordingly, increasing the safety standards for many dike rings in the Netherlands. This also means a significant increase in expenses for protection against flooding in the Netherlands in the near future.

We also note that this problem and model are relevant for many other deltas in the world that are threatened by floods. This growing threat has been described extensively in a recent paper (Syvitski et al. (2009)). The paper presents an assessment of 33 important deltas across the world, noting that: “in the past decade, 85% of these deltas experienced severe flooding, resulting in the temporary submergence of 260,000 km^2. Moreover, it is conservatively estimated that the delta surface area vulnerable to flooding could increase by 50% under the current projected values for sea level rise in the twenty-first century. Close to half a billion people live on or near deltas, often in megacities. Twentieth-century catchment developments, and population and economic growth have had a profound impact on deltas. As a result, these environments and their populations are under a growing risk of coastal flooding, wetland loss, shoreline retreat and loss of infrastructure. More than 10 million people a year experience flooding due to storm surges alone.".

Concerning the main mathematical contributions, we mention that we corrected and extended Van Dantzig's model and solution in several ways. Moreover, for a class of the investment function we were able to prove that there is a periodic solution that satisfies the first order conditions, and we derived formulas to calculate this solution. For other choices of the investment cost function we developed a Dynamic Programming approach.

5.2 Further research

It is an interesting topic for further research to prove that the periodic solution derived for the exponential cost function is really the global optimum. There is much numerical evidence that this is the case, but a mathematical proof is still missing.

In the model we assume that the dike ring is homogeneous, i.e. that the characteristics for, for instance, the flood probability and the investment costs are the same for all dikes in the dike ring. However, this does not necessarily apply for many of the dike rings in the Netherlands, since in practice there are dike rings with up to 10 different dike segments. For such cases the number of possible states in the Dynamic Programming approach explodes, and for this a different approach is proposed in Brekelmans et al. (2012).

The model, method and theory developed in this paper may also be used for other optimization problems that have a similar structure. This is a subject for further research, with a particular focus on maintenance optimization problems.

Several parameters in the model are also uncertain, especially the parameters α, γ, η, and P_0^-. In Brekelmans et al. (2012) we propose a regret approach to obtain solutions that are robust against uncertainty in these parameters.

It is worthwhile analyzing the effect of partitioning a certain dike ring area by what is termed a 'partitioning dike'. The model needs to be adapted to study the effect of such a partitioning.

It would also be extremely interesting to apply our model to other deltas, such as New Orleans.

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