Identifying Sorting – In Theory[*]

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Abstract

Assortative Matching between workers and firms provides evidence of the complementarities or substitutes in production. We argue that using wage data alone, it is virtually impossible to identify whether Assortative Matching is positive or negative. For every production function that induces positive sorting we can find a production function that induces negative sorting, whilst generating identical wages. Even though we cannot identify the *sign* of the sorting, we can identify the *strength*, i.e., the magnitude of the cross-partial, and the associated welfare loss. We show that a worker's wages are non-monotonic in firm type. This is due to the fact that in a sorting model, wages reflect the opportunity cost of mismatch. This non-monotonicity allows us to establish analytically that standard fixed effects regressions are not suitable to recover the strength of sorting. We propose an alternative procedure that measures the strength of sorting in the presence of search frictions independent of the sign of the sorting. Knowing the strength of sorting enables us to measure the efficiency loss due to mismatch.

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1 Introduction

Sorting of workers to jobs matters for the efficient production of output in the economy. If there are strong complementarities or substitutes between workers and jobs, the exact allocation has large efficiency implications. In contrast, when complementarities are nearly absent, not much output is lost from the random allocation of workers to jobs. This is important for policy, for example whether we want to design an unemployment insurance program that provides incentives for workers to look for the "right" job instead of accepting the first offer (see for example Acemoglu and Shimer 1999). Complementarities and sorting between worker skills to firms' technological ability also have profound implications for wage inequality across differently skilled workers (see for example Sattinger 1975). Sorting based on complementarities is the driving force in a variety of applications: it is central to the argument of skill-biased technological change; it affects the impact of immigration on the domestic labor force; and it shapes the effect of subsidies to education.

Given the importance of sorting, a large body of recent empirical literature has estimated whether sorting is positive or negative. In part, this renewed interest has been catalyzed by the availability of worker-firm match data (see Postel-Vinay and Robin (2006) for an overview). Several empirical papers find an insignificant or even negative correlation in fixed effects between worker and firm types. This result has been replicated for a number of countries including France, US, Denmark and Brazil. The result is taken as indication that Positive Assortative Matching between workers and firms does not play a major role in the labor market.

On the other hand, recent work reveals that simulations of models with strong complementarities and sorting nonetheless generate small or even negative correlations of the simulated fixed effects of workers and firms. Lise, Meghir and Robin (2008), Lopes de Melo (2008), and Bagger and Lentz (2008) study variations of structural labor search models with an infinite horizon, and simulate as well as estimate those models with matched employer-employee data. Our objective is to provide a much simpler framework, but one that allows us to investigate theoretically how to measure the extent of sorting and to derive the correlation between fixed effects analytically.

We obtain the following results. First, in the frictionless matching model of a competitve labor market (Becker 1973), we show that identifying whether sorting is positive or negative is impossible using wage data alone.[1] To see this, note that in this model more productive workers always earn higher wages and more productive firms always make more profits. Under Positive Assortative Matching (PAM), the more productive firm also has the highest marginal product from labor, i.e., the cross-partial is positive. This implies that high type firms hire high type workers and they pay high wages. Under Negative Assortative Matching (NAM) instead, low type firms have a comparative advantage from hiring more productive workers, i.e., the cross-partial is negative.[2] As a result, high type firms pay lower wages. By ranking firms according to the wages they pay, we do not identify the most productive

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[1]Most people's prior is that production technologies exhibit PAM. Yet, NAM is not inconceivable: high productivity firms might have invested in skill-reducing technologies, in which case the marginal benefit from hiring a better worker is lower (e.g., Wallmart). Also, we should point out that a lower marginal benefit (i.e., a negative cross-partial) does not mean that higher types generate less value, but rather that the increase in output due to a better worker is not as large.

[2]We use the term comparative advantage to denote a larger absolute gain in output from matching with a better worker, rather than the stronger concept of a larger percentage gain as used e.g., in Sattinger (1975).

firm. Without any additional data on the profitability of each individual job, it is impossible to identify whether sorting is positive or negative. This is true even if we consider wages off the equilibrium path.

Second, we explicitly consider wages away from the Beckerian equilibrium allocation, due to mismatch for example as a result of search frictions. We find that the first-order effect is that wages of a given worker have an inverted U-shape around the optimal allocation, the "bliss point". This non-monotonicity reflects the *opportunity cost* of a firm to match with an inappropriate worker type. For a given worker, wages are low if he matches with a "bad" firm, because the value that is generated is low. Maybe less obviously, his wage is also low if he matches with a very "good" firm. The reason is that higher productivity firms have to be compensated for their willingness to match with a "bad" worker because it destroys their opportunity to match with a "good" worker. Under complementarities that firm has a disproportionately larger marginal product with the good worker. This leads to the bliss-point in the pattern of compensation if a worker meets the "right" firm, rather then a wage schedule that is increasing everywhere in the type of firm. It is well known that this non-monotonicity is a crucial feature of the competitive wage setting in Becker's original theory, and that it remains important under bargaining in the presence of explicit frictions. In an infinite horizon search model, the non-monotonicity was pointed out by Gautier and Teulings (2006) who take this into account when estimating the cost of search frictions of a technology that is assumed to be log-supermodular.

The net effect on a given worker's wages from increasing the firm type is therefore ambiguous and second-order. For the simplest, type-independent cost of delay (also studied in Chade (2001) and Atakan (2006)), we explicitly show that the net effect is equal to zero under some common specification. This version is a close reformulation of Becker (1973). When search costs are type-dependent (as in Shimer and Smith (2000) for example), the net effect may still be either positive or negative. The additional component induced through type-dependence is proportional to the magnitude of the friction and, compared to the inverted U-shape, the net effect of the firm type on wages is second-order and difficult to isolate in the data.

Third, and in spite of the fact that we cannot identify the *sign*, we develop a method that in theory allows for identification of the *strength* of sorting. Ultimately, efficiency properties depend on how big the complementarities/substitutes are. Identification derives from the distinct features of the range of wages a worker receives who has been observed repeatedly. First, we derive from the range of wages paid what the cost of search is. The highest observed wage corresponds to her bliss-point and we use this to order the workers and obtain the type distribution. Likewise, we can obtain an order of the firms by the level of wages that they pay. The difference between the highest and the lowest wage is corresponds to the cost of search. Second, given the search cost, the fraction of the firm population that an agent is willing to match with identifies the strength (in absolute value) of the cross-partial of the production function. This is possible because the strength of the cross-partial directly reflects the efficiency loss due to mismatch.

The setup of our economy is very simple with dynamics reduced to two periods. The objective is to solve analytically what the standard infinite horizon models cannot deliver. This makes the models very specific, but we argue that our insights extend to the fully dynamic steady state models. In section 5 we discuss alternative models and argue that whenever wages reflect the competitively determined outside option, they must necessarily be non-monotonic in firm type.

Before proceeding to set up the model, we briefly lay out the empirical issue. In their seminal paper, Abowd, e.a. (1999) use matched employer-employee data to decompose wages into different effects related to worker and firm characteristics. With unrestricted correlation among the effects they are able to estimate the firm and worker components of wages. The generality of their econometric approach allows for many different estimation techniques to determine each of the components. A by-product of their contribution is a simple empirical measure of sorting that can be obtained by estimating a log-wage equation in which wages are a function of a worker fixed effect, a firm fixed effect, and an orthogonal error term: $\log w_{it} = a_{it}\beta + \delta_i + \psi_{j(i,t)} + \varepsilon_{it}$, where w_{it} denotes the wage, a_{it} are time varying observables of workers, δ_i is a worker fixed effect, ψ_j is the fixed effect of the firm j at which worker i is employed at time t, and ε_{it} is an orthogonal residual. That is, ψ_j captures the average effect that a firm has on the wages of the workers that are willing to match with it. The correlation between δ_i and ψ_j in a given match is taken as an estimate of the degree of sorting.

In a Beckerian theory of wage setting, wages reflect the outside options of the firms and therefore they are non-monotonic in firm type.[3] We show in our theoretical exercise that the assumption that the firm effect is independent of the worker's type is theoretically not justified in this setting. In particular, for those workers who are matched with a firm that has a lower rank than their own, the wage increases when the firm type increases because the worker-firm "fit" improves. In contrast, workers who are matched with a higher ranked firm see a decrease in the wage when the firm becomes better because the worker-firm "fit" deteriorates. This implies that the correlation between the firm and worker fixed effect cannot be taken as a measure of sorting. We show this more directly by deriving the fixed effect theoretically and showing that the positive wage effects that a better firm has on some workers can indeed cancel exactly with the negative effects on other workers, rendering the fixed effect approach inable to detect sorting.

A similar logic holds in the context of matching with frictions. With wage bargaining in a model with search frictions, the set of eligible partners is bounded by those matches where the match surplus is zero relative to the value of continued search. For all acceptable partners the surplus is positive. For a given worker type, the surplus goes to zero for a match both with too bad a firm and too good a firm. Any bargaining procedure that pays wages that are monotonic in the surplus after accounting for the outside option of attracting a more appropriate type will therefore result in wages being non-monotonic in firm type.

The goal of our analysis is to lay out this logic in the simplest possible environment. First, we consider the frictionless benchmark, and then we extend it in a straightforward way to a model with frictions that allows worker and firm mismatch. Frictions are modeled in a two-stage set up: a stage of random matching is followed by a frictionless matching stage. The benefit of our modeling approach is that the main effects that drive the wage determination become clearly visible and highlights the forces, limitations, and possibilities that arise in estimations that are based solely on wage data. As such, it informs our understanding of the results obtained in more complicated infinite horizon steady-state models that preclude closed-form theoretical analysis but are often used for structural estimation.

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[3]Abowd, e.a. (2004) propose a bargaining procedure in which higher firms pay higher wages in line with the regression in the text as a test of Becker's (1973) idea of matching. We explain in section 5 that such a pay schedule is inconsistent with Becker's theory because every worker would like to match with the best firm. Becker's theory is inseparably linked to a theory of wages which prevents such overcrowding of workers at top firms.

2 The Frictionless Model

In order to make our point we start with the following simple matching model following Becker (1973). There is a unit mass of workers and a unit mass of firms. Workers and firms are heterogeneous in terms of their productivity. Workers draw their type x from distribution $\Gamma(x)$ with smooth density $\gamma(x)$ on $[0,1]$. Firms draw their type y from distribution $\Upsilon(y)$ with smooth density $\upsilon(y)$ on $[0,1]$.

When types x and y form a match, they produce positive output $f(x,y) \geq 0$ whilst having an outside option of remaining unmatched. We assume that workers and firms can be ranked in terms of their productivity, i.e., $f_x > 0$ and $f_y > 0$. Then it is without loss of generality to index a worker by his *rank* in terms of productivity, i.e., by the fraction of workers that are less productive then him. Similarly, we can identify each firm by its rank in the distribution of firm productivities. This means that $\Gamma(\cdot) = \Upsilon(\cdot) = x$, i.e., the distributions are uniform.[4] Assume that workers who do not get matched obtain a payoff of zero, and since output is non-negative, all agents will prefer to match.

For the assignment of workers to firms the cross-partial of the production function is important. We do not restrict the sign of the cross-partial since this will be instrumental in determining whether there is positive or negative assortative matching. Denote by $\mathcal{F}$ the class of all functions f that are monotonic: $f_x, f_y > 0$; and that have a monotonic marginal product: $f_{xy}(x,y)$ is either always positive or always negative.[5] The assumption that the cross-partial does not change sign allows us to unambiguously talk about positive or negative sorting. Production functions with complementarities ($f_{xy} > 0$) are in set $\mathcal{F}^+ \subset \mathcal{F}$. Production functions with substitutes ($f_{xy} < 0$) are in set $\mathcal{F}^- \subset \mathcal{F}$.

To illustrate the implications of our analysis we will derive our results for the following examples of production functions

$$f^+(x,y) = \alpha x^\theta y^\theta + h(x) + g(y), \tag{1}$$
$$f^-(x,y) = \alpha x^\theta (1-y)^\theta + h(x) + g(y), \tag{2}$$

where $g(\cdot)$ and $h(\cdot)$ are increasing functions and $\alpha \geq 0$ and $\theta > 0$ are parameters that indicate the strength of the complementarities. We assume that $g(y)$ is such that higher firms produce higher output even under the second specification. It is obvious that $f^+ \in \mathcal{F}^+$ and $f^- \in \mathcal{F}^-$.

An assignment of workers x to firms y is denoted by μ, i.e., $\mu(x) = y$ means that worker x gets hired by firm y. In this part of the paper we assume a competitive matching market. A market equilibrium specifies an assignment between x's and y's and some wage schedule $w(x,y)$ that determines the split of output between the worker and the firm. The payoff to the worker is $w(x,y)$ and the payoff to the firm is $\pi(x,y) = f(x,y) - w(x,y)$. Both workers and firms take the wage schedule as given. The tuple of functions (μ, w) is an equilibrium if no worker wants to switch to a different firm at the market wages,

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[4]The uniformity assumption is without loss of generality since the production function can be identified only up to a normalization: For a general $\Gamma(\cdot)$ and $\Upsilon(\cdot)$ and $f(\cdot, \cdot)$ we can alternatively consider a uniform type space in which only the ranking matters and an alternative production function $\tilde{f}(x,y) = f(\Gamma(x), \Upsilon(y))$.

[5]Later, in section 6 we discuss the virtues of relaxing this assumption.

and no firm wishes to employ a different worker,[6] i.e.,

$$w(x, \mu(x)) \geq w(x, y) \text{ for all } x \text{ and } y$$
$$\pi(\mu^{-1}(y), y) \geq \pi(x, y) \text{ for all } x \text{ and } y,$$

and no agent prefers to remain single, i.e., $w(x, \mu(x)) \geq 0$ for all x and $\pi(\mu^{-1}(y), y) \geq 0$ for all y.

We derive the main prediction of Becker's (1973) model concerning the wages in the economy. In equilibrium, each firm y maximizes profits, taking the wage schedule as given:

$$\max_x f(x, y) - w(x, y).$$

This yields the first order condition

$$f_x(x, y) - \frac{dw(x, y)}{dx} = 0. \tag{3}$$

Let $w^\star(x)$ be the equilibrium wage of worker x. Integrating (3) along the equilibrium path yields

$$w^\star(x) = \int_0^x f_x(\tilde{x}, \mu(\tilde{x})) d\tilde{x} + w_0, \tag{4}$$

where the constant of integration $w_0 \in [0, f(0,0)]$ can be thought of as some exogenous bargaining rule that splits the surplus between the lowest types in case these types have a positive surplus over remaining single.[7] Observe that the worker obtains exactly his marginal product along the equilibrium allocation. Therefore, equilibrium profits of type y are given by output minus the wage $w^\star$ with the optimal worker $\mu^{-1}(y)$. This can be re-written as

$$\pi^\star(y) = \int_0^y f_y(\mu^{-1}(\tilde{y}), \tilde{y}) d\tilde{y} + f(0,0) - w_0 \tag{5}$$

Furthermore, we know from Becker's analysis that matching is positive assortative when the production function is supermodular ($f_{xy} > 0$), in which case $\mu(x) = x$. Under submodularity ($f_{xy} < 0$) in equilibrium the matching is negative assortative and $\mu(x) = 1 - x$, since lower type firms have a higher *marginal* value for better workers and are willing to pay more for them. With this in mind, we show that in this simple competitive model the sign of sorting – i.e., the sign of the cross-partial – cannot be identified from wage data. We first show this result on the equilibrium path, then we show it also holds off the equilibrium path. In the next section we build an extended model with search frictions where the wages off the equilibrium path actually arise.

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[6]It is well-known that a strict cross-partial yields a one-to-one mapping $\mu(\cdot)$ in equilibrium. In general $\mu(\cdot)$ is a correspondence, with the equilibrium definition extended to all pairs in that correspondence.

[7]When $f(0,0) = 0$, then $w_0 = 0$ and the wage schedule is uniquely determined. Otherwise there are a continuum of competitive equilibria associated with different w_0, and we assume that the specific split w_0 is a primitive determined by some exogenous bargaining rule.

2.1 On the equilibrium path

We will first illustrate the result by considering our restricted class of production functions outlined above and then present the general theorem. Suppose the underlying production technology is not known and the true technology is either one of the two example technologies f^+ given in (1) or f^- given in (2). By (4) the wages under f^+ and f^- are

$$w^{\star,+}(x) = \int_0^x f_x^+(\tilde{x}, \tilde{x})d\tilde{x} + w_0 = \frac{\alpha}{2}x^{2\theta} + h(x) - h(0) + w_0$$

$$w^{\star,-}(x) = \int_0^x f_x^-(\tilde{x}, 1 - \tilde{x})d\tilde{x} + w_0 = \frac{\alpha}{2}x^{2\theta} + h(x) - h(0) + w_0.$$

Under both technologies the wages on the equilibrium path are exactly identical, and from wage data alone one cannot distinguish between positive and negative sorting. The problem is obtaining the order of the firms. If we only have wage data and no profit data, and we derive the order on the firms by ranking them by increasing wages, we will obtain two different orders depending on whether we have complements or substitutes. To see this, observe that under positive assortative matching (henceforth PAM) higher type firms pay higher wages along the equilibrium path whereas under negative assortative matching (NAM) higher type firms pay lower wages. In the former $w(y, y) = \frac{\alpha y^{2\theta}}{2}$ is increasing in y, in the latter $w(1 - y, y) = \frac{\alpha(1-y)^{2\theta}}{2}$ is decreasing in y. This result is true for any general production technology as summarized in the proposition that follows below.

Figure 1 shows the profits, wages and total output when $f^+ = xy + y$ and $f^- = x(1 - y) + y$. Observe that wages are identical in both cases (blue), but that profits of the firm matched to worker x are decreasing in worker type x under f^-. While higher y firms have higher profits, higher x workers are matched with lower y firms who obtain lower profits. That is, even under NAM $\pi^{\star,-}(y) = y + \frac{(1-y)^2}{2}$ is increasing in y even though $\pi^-(x, \mu(x))$ is decreasing in x. The payoffs under both technologies are summarized in the following table.

	$f^+ = xy + y$	$f^- = x(1 - y) + y$
$w(x, \mu(x))$	$\frac{x^2}{2}$	$\frac{x^2}{2}$
$\pi(x, \mu(x))$	$\frac{x^2}{2} + x$	$\frac{x^2}{2} + 1 - x$
$f(x, \mu(x))$	$x^2 + x$	$x^2 + 1 - x$

Proposition 1 *For any production function $f \in \mathcal{F}^+$ that induces positive sorting there exists a production function $f \in \mathcal{F}^-$ that induces negative sorting and the equilibrium wages $w^*(x)$ are identical under both production functions.*

Proof. From equation (4) we obtain the wage schedule. When generated by an underlying production process that is supermodular f^+ this is

$$w^{\star,+}(x) = \int_0^x f_x^+(\tilde{x}, \tilde{x})d\tilde{x} + w_0^+,$$

and for a submodular process f^- it is

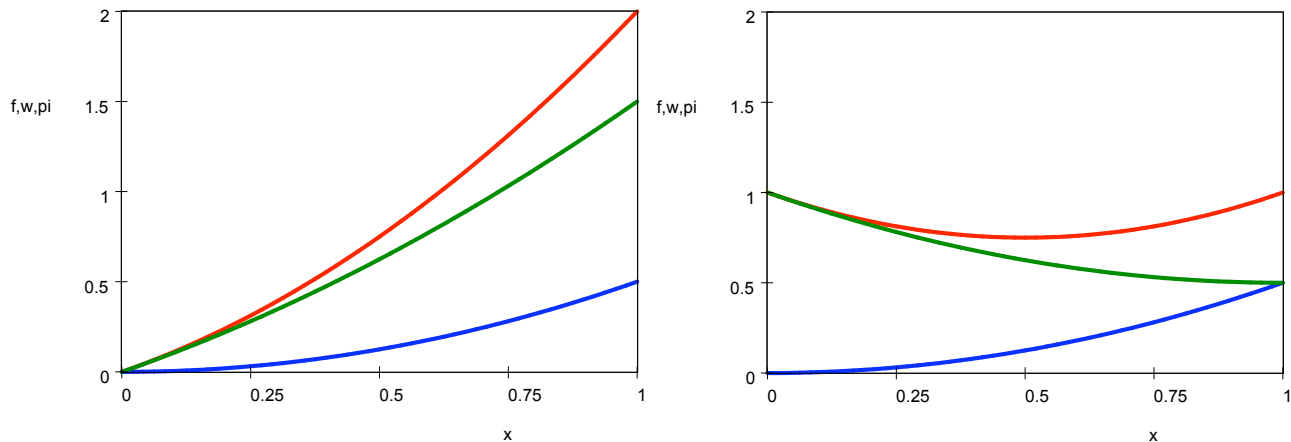$$w^{\star,-}(x) = \int_0^x f_x^-(\tilde{x}, 1 - \tilde{x})d\tilde{x} + w_0^-.$$

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Figure 1: Equilibrium Wages $w(x, \mu(x))$, Profits $\pi(x, \mu(x))$, Total Output $f(x, \mu(x))$ under $f^+ = xy + y$ [left] and $f^- = x(1-y) + y$ [right] with $w_0 = 0$.

Observe that since $w^{\star,+}(0) = w_0^+$ and $w^{\star,-}(0) = w_0^-$, both wage schedules can be identical when the free bargaining paramter satisfies $w_0^+ = w_0^- = w_0$. Then we obtain $w^{\star,+}(x) = w^{\star,-}(x)$ for all x if $f_x^+(\tilde{x}, \tilde{x}) = f_x^-(\tilde{x}, 1 - \tilde{x})$. For any $f^+(x, y)$ on $[0,1]^2$ we can define $f^-(x, y) = f^+(x, 1 - y)$ on $[0,1]^2$. The only restriction is that this function may not be increasing in y, so we may need to "augment" the function to ensure that f_y is positive. If f_x, f_y are bounded, it is sufficient to add a term $\tau \cdot y$ where $\tau > 0$ is large enough to ensure $f_y > 0$ everywhere. If f_y is not bounded and negative, we need to add a function $g(y)$ that increases faster than the decrease of $f^+(x, 1 - y)$ in y. ∎

2.2 Off the equilibrium path

Identification needs variation. Identification of sorting from equilibrium wages may be difficult simply because there is no independent variation across firms and workers. In the frictionless case workers sort perfectly in the sense that each type of firm attracts exactly one worker type. Even if workers became unemployed and could match again later without frictions, the panel dimension would not allow us to identify a separate effect for firms and workers, because workers will always end up in the same type of firm. There would not be any wage variation, and it cannot be identified whether a high wage is due to the worker ability or the firm productivity.

Here we entertain the idea that workers "tremble" to off-the-equilibrium firms and obtain the associated off-equilibrium wages. If workers tremble, we would be able to see wage variation for workers and firms of the same type. In the next section, we extend our environment to include search frictions such that those variation actually arises in equilibrium. Here we argue that off-equilibrium wages are still not informative about the sorting in the market, since any procedure that identifies firm types y from wages alone for a technology in $\mathcal{F}^+$ will misidentify firm types as $\hat{y} = 1 - y$ for some technology in $\mathcal{F}^-$. Equally important is that the analysis of off-equilibrium wages highlights the key competitive reason why a given worker may not earn higher wages at more productive firms even under $\mathcal{F}^+$.

The wage schedule off equilibrium in the frictionless model is such that neither firms nor workers

would want to deviate to such matches. Off the equilibrium path this wage schedule is not uniquely pinned down and wages range between the lowest wage that is just high enough to prevent firms from deviating and the highest wage that is just high enough to prevent workers from deviating. While the matched agents' wages $w(x, \mu(x))$ are determined as above, the wages of the mismatched agents $w(x, y)$ must satisfy:

$$f(x, y) - w(x, y) \leq \pi(\mu^{-1}(y), y) \tag{6}$$
$$w(x, y) \leq w(x, \mu(x)) \tag{7}$$

where $\mu(x) = y$ in the case of PAM and $\mu(x) = 1 - y$ in the case of NAM. For a given (x, y) combination, call the set of wages that are consistent with (6) and (7) $W(x, y)$.

It is important to note the implication of (7): wages are highest at the firm that is most appropriate for the worker. Even under positive assortative matching, a worker x who tries to trade with a firm that is higher (or lower) than his optimal type $\mu(x)$ will earn lower wages. At less productive firms this arises for the obvious reason that the surplus is too low. At more productive firms this arises because the firm forgoes the benefit of hiring the more appropriate worker and has to be compensated for this opportunity cost. In the extreme this can even lead to negative wages at very high firms, because the output when hiring worker x is so much lower than hiring a higher worker type. The standard idea that competitive wage setting takes into account the opportunity costs of hiring an inappropriate worker leads to a non-monotonicity in the wage schedule. We will see this effect even in the mismatch model in the next section, and it will be the main reason why the fixed effects approach will not identify sorting.

Before analyzing the non-monotonicities further, we briefly highlight the inability of determining the sign of sorting even at off-equilibrium wages. It is easily verified that in the case of the technology $f^+(x, y)$ any wage $w(x, y)$ in $W(x, y)$ satisfies

$$\alpha (xy)^\theta - \frac{\alpha}{2} y^{2\theta} \leq w(x, y) - h(x) \leq \frac{\alpha}{2} x^{2\theta}. \tag{8}$$

In the case of $f^-(x, y)$ any wage $w(x, y)$ in $W(x, y)$ satisfies

$$\alpha x^\theta (1 - y)^\theta - \frac{\alpha}{2} (1 - y)^{2\theta} \leq w(x, y) - h(x) \leq \frac{\alpha}{2} x^{2\theta}, \tag{9}$$

which is identical to (8) if we misinterpret the types as $\hat{y} = 1 - y$. If we have no information on profits, as before we cannot derive the order on y simply from wage data, not even after observing off-the-equilibrium path wages. The bounds on the wages under PAM and NAM are identical if we use the order on wages to derive the order on y (in which case under NAM we assign the order $1 - y$ to the firms). The static Beckerian model will therefore not allow for identification of assortative matching based on wage data alone.

Proposition 2 *For any production function $f \in \mathcal{F}^+$ that induces positive sorting there exists a production function $f \in \mathcal{F}^-$ that induces negative sorting and the equilibrium wage sets $W(x, y)$ in the former are identical to equilibrium wage set $W(x, 1 - y)$ in the latter.*

Proof. The proof follows the same argument as in Proposition 1. ∎

3 Mismatch due to Search Frictions

We now consider an extended model with mismatch due to frictions caused by delay. Unlike the static Beckerian model, search frictions may induce different behavior in the acceptance decision of matches. First, assuming that the technology is generated by a supermodular production technology, we derive the equilibrium allocation in the presence of a type-independent search cost. We address the issue of identification of positive/negative sorting in this model, and whether we can identify sorting from wage data alone. Second, for this model we derive from the theory the firm fixed-effect. The case of type-independent search costs considerably simplifies the analysis, but the gist of the argument carries over for a more general search cost. Below in section 5 we consider a model with type-dependent search cost.

3.1 Type-Independent Search Costs Under Supermodularity

Our model has hiring in two stages. In stage one, each worker is paired with one firm. The pairings are random. One can think of this part of the hiring process as standing in for some connections that workers have to the labor market prior to engaging in an extensive search for labor. The pair can either agree to stay together at some wage, or search for a better partner. Those who decide not to stay together and those who did not get paired each incur a search cost c due to the delay. In the second stage, all remaining agents are matched according to the competitive, frictionless allocation as outlined above.[8] After the search process has ended production starts. To allow for a panel dimension in the observations (i.e., over time, each worker matches with more than one firm, and each firm matches with more than one worker) we assume that each agent goes through this two-stages hiring process several times in his life.[9]

We consider the same class of production functions $\mathcal{F}$. For exposition it will be convenient to restrict the supermodular function in $\mathcal{F}^+$ to functions with symmetric cross-partial such that $f_{xy}(x,y) = f_{xy}(y,x)$. For submodular functions in $\mathcal{F}^-$ which induce negative sorting it will be convenient to restrict attention to symmetry of the form $f_{xy}(x,y) = f_{xy}(1-y, 1-x)$.

We assume that the transfer in the first period is determined by Nash bargaining with equal bargaining weights. We illustrate this with our example production function f^+ in (1). When a worker x meets a firm y, the payoff from matching is $f(x,y)$. Waiting until next period and matching in the perfectly competitive labor market yields payoff $w(x, \mu(x)) - c$ to the worker and $\pi(\mu^{-1}(y), y) - c$ to the firm. A first-period match will therefore be accepted provided that the current match surplus over

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[8]Here we need to worry about the possibility that when the acceptance sets span the entire type space (e.g., because of high search costs or low complementarities), no agents are left in the second stage, in which case the continuation payoff is not determined. Without modeling this explicitly, we think of a tremble that ensures that there are always some agents who end up in the next period. For many parameters each worker and firm type rejects some agents on the other side of the market, and these agents will indeed move to the second stage.

[9]One can think of termination of all jobs after a specified time period, with a new period of matching that follows. More generally one may think of this as standing in for exogenous separations into a steady state matching model.
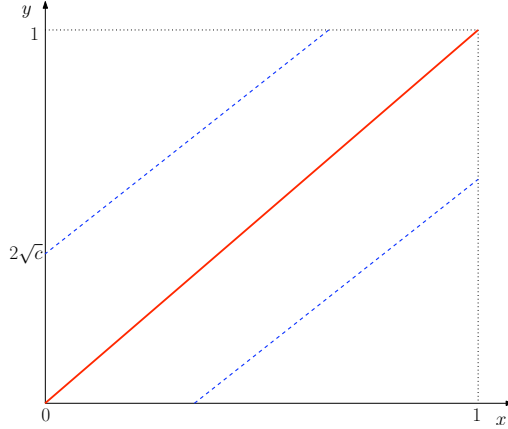
Figure 2: Acceptance sets with type-independent search costs for $\theta = 1$.

waiting is positive,[10] i.e.,

$$f(x,y) - (w^\star(x) + \pi^\star(y) - 2c) \geq 0. \tag{10}$$

For a given firm y we call the set of worker types that fulfill (10) his acceptance set and denote it by $A(y)$ for the firms and by $B(x)$ for the workers.[11] Similar to the work by Atakan (2006) we can show that the bounds of this set are increasing if the production function is in $\mathcal{F}^+$ and decreasing if it is in $\mathcal{F}^-$, which naturally extends the notion of sorting to sets.

For our example production function f^+ in (1) it is easy to verify that (10) reduces to

$$\alpha(xy)^\theta - \frac{\alpha}{2}x^{2\theta} - \frac{\alpha}{2}y^{2\theta} \geq -2c. \tag{11}$$

and therefore the acceptance set becomes

$$A(y) = \left[(y^\theta - 2\sqrt{c/\alpha})^{1/\theta}, (y^\theta + 2\sqrt{c/\alpha})^{1/\theta} \right]. \tag{12}$$

The matching sets are illustrated in Figure 2 for the case $\theta = 1$. Due to symmetry the acceptance set of the workers looks identical.

Because the surplus is divided equally, the worker obtains half of this surplus on top of his outside

---

[10]It may well be that for low types the surplus in the next period does not exceed the total waiting cost of $2c$. In order to avoid keeping track of endogenous entry, we assume that people will search even if that is the case. This may be due to the fact that the outside option (e.g. unemployment benefits) are contingent on searching. This issue never arises when $f(0,0) > c$ and $w_0 > c$, as all agents than have an incentive to search, or when search costs are proportional as in Section 6.

[11]Note that for supermodular functions in $\mathcal{F}^+$ this acceptance set is identical for workers and firms of the same type types, which is a general consequence of the symmetry of f_{xy}. Therefore the distribution of types in the second stage is identical for workers and firms, and therefore the equilibrium assignment is still $\mu(x) = x$. Similarly, for submodular functions in F^- is can be shown that under our symmetry condition when x accepts y then $1 - x$ accepts $1 - y$, which leads to distributions that are symmetric around $1/2$ $(1 - \Gamma(x) = \Gamma(1 - x), 1 - \Upsilon(y) = \Upsilon(1 - y))$ and the equilibrium assignment indeed remains $\mu(x) = 1 - x$.

option that is given by his value of waiting. Therefore, his wage is

$$w(x, y) = \frac{1}{2}\left[f(x,y) - w(x, \mu(x)) - \pi(\mu^{-1}(y), y) + 2c\right] + w(x, \mu(x)) - c \qquad (13)$$

$$= \frac{1}{2}\left[f(x,y) + w(x, \mu(x)) - \pi(\mu^{-1}(y), y)\right]. \qquad (14)$$

It is straightforward to see that this wage is exactly in the middle of the acceptance set $W(x, y)$ outlined in (6) and (7) for off-equilibrium-wages of the frictionless model. This makes this model very attractive to work with. It translates the insights from Becker's (1973) assignment model to a model of mismatch. It immediately implies that positive and negative sorting cannot be identified by observed wage data, because the wages under supermodular production functions coincide with those under submodular production functions (under misinterpretation of the unobserved firm type). Since the wage distributions coincide, the identification cannot come from the acceptance decisions (i.e., the speed of matching) between first and second period, either, because identical wages imply identical acceptance decisions (again under the misinterpretation of firm types).[12]

Proposition 3 *For every supermodular production function $f^+ \in \mathcal{F}^+$ that induces positive sorting there is a submodular production function $f^- \in \mathcal{F}^-$ that induces exactly the same wages for workers when we reinterpret firm types as $\hat{y} = 1 - y$.*

Proof. Wages in the second period coincide by Proposition 1. Wages in the first period coincide because they are in the exact arithmetic middle of the wage set $W(\cdot, \cdot)$ which by Proposition (2) coincide under the re-interpretation. ∎

For our example production technology f^+ we get as wages

$$w(x, y) = \frac{\alpha}{2}(xy)^\theta + \frac{\alpha}{4}x^{2\theta} - \frac{\alpha}{4}y^{2\theta} + h(x).$$

For some of the results it will be instructive to rewrite the wages as a function of the distance k between the worker and the firm, which for the special case of $\theta = 1$ becomes particularly tractable:

$$w(x, x - k) = w(x, x + k) = \frac{\alpha}{2}x^2 - \frac{\alpha}{4}k^2 + h(x).$$

This shows that a worker has a bliss point when matching with a firm with identical type, and looses quadratically with the distance to the firm. The reason is that a worker who matches with a firm that has too low a type does not produce a lot of output. On the other hand, a worker who wants to induce a much better firm to match with him has to compensate the firm for not matching with a more appropriate worker. Therefore a worker is not necessarily better matching with a higher type firm. In a large region – i.e., whenever the firm is higher ranked than the worker – wages fall by matching with even better firms. Figure 3 illustrates the wage schedule of a worker as a function of the distance to the firm he matches with, and highlights the fact that the wage falls in firm type in part of the region. This result holds more generally

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[12]Bargaining weights different from 1/2 or different costs for workers and firms will change the bargaining sets and the wages, but it is straightforward to show that they will not affect our results on identification.
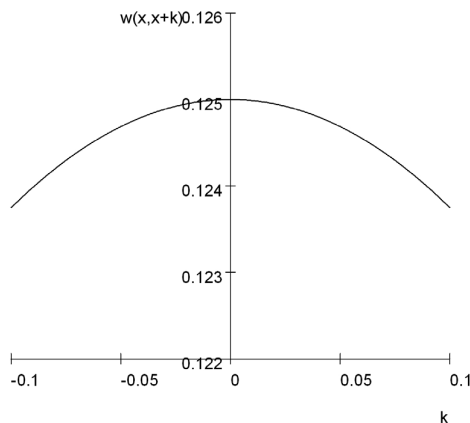
Figure 3: First period wages for under mismatch with a type $y = x + k$ that is k away from the bliss point. [Graph for $x = .5$, $c = .25$, $\alpha = \theta = 1$, $h(x) = 0$.]

Proposition 4 *(Bliss Point). For each $x \in (0,1)$ wages $w(x,y)$ are non-monotone in y.*

Proof. Wages $w(x,y)$ are in the exact arithmetic middle of the wage set $W(x,y)$. Since worker type x chooses the optimal wage, by (7) any wage in $W(x,y')$ is lower than the wage $w(x,\mu(x))$ under the optimal assignment. Since $W(x,\mu(x)) = w(x,\mu(x))$ the optimal wages arise in the first stage. Because search costs are positive, all firm types close to $\mu(x)$ have a positive surplus and thus will form a match with x in the first stage. Therefore, wages are non-monotone around the optimum wage $w(x,\mu(x))$. ∎

Note that none of the results depend on an equal split of the surplus or identical search costs for workers and firms. While this specification makes the exposition especially tractable because we can immediately rely on the results of the previous section, a bargaining power $\gamma \in (0,1)$ for the workers and a search cost c_w for workers and c_f for firms would only linearly rescale the wages without affecting the results further.[13]

In the following we show that this non-monotonicity of the wage schedule limits a fixed effect estimator to detect the sign and the strength of sorting.

3.2 Inconclusive Firm-Fixed-Effects

In this section we assess the ability of the fixed effects approach that we discussed in the introduction to detect the degree of sorting. We only consider the degree of sorting because we know from the previous analysis that the sign of sorting cannot be distinguished. We show that the decreasing part of the wage schedule translates into ambiguous fixed effects.

Fixed effect estimation relies on the panel dimension of matched employer-employee data sets. We assume that the 2-stage hiring process outlined above repeats T times for each agent. That is, every worker has T spells of unemployment, in each of which she goes through the hiring process to obtain a new job. For this theoretical exposition we assume that T is large so that the realized wages of each

---

[13]The wage equation in (14) would simply change to $w(x,y) = \gamma \left[f(x,y) + w(x,\mu(x)) - \pi(\mu^{-1}(y), y) \right] - (1-\gamma)c_w + \gamma c_f$.

worker reflect the distribution of wages that she faces. For a given production function f and type distributions $\Gamma(\cdot)$ and $\Upsilon(\cdot)$ for workers and firms, we can decompose the resulting first period wages according to the fixed effects approach. In the fixed effect approach, the wage when worker x matches with firm y is given by the sum of the worker's fixed effects $\delta(x)$ and the firm's fixed effect $\psi(y)$ plus a residual. We can write

$$w(x,y) = \delta(x) + \psi(y) + \varepsilon_{xy}, \tag{15}$$

where ε_{xy} is the residual. We require the fixed effects to be unbiased. That is, the fixed effects $\delta(x)$ and $\psi(y)$ are determined such that the residual is zero in expectation for each x (when integrated over the y's she is matched to) and for each y (when integrated over the x's the firm is matched with). In the example below this also ensures that the sum of squared errors accross all observations is minimized.[14]

In the appendix we show that the fixed effects are unbiased when

$$\delta(x) = \int_{B(x)} [w(x,y) - \psi(y)]\, d\Upsilon(y|x), \tag{16}$$

$$\psi(y) = \int_{A(y)} [w(x,y) - \delta(x)]\, d\Gamma(x|y), \tag{17}$$

where $A(y)$ and $B(x)$ are the acceptance sets, and $\Upsilon(y|x)$ and $\Gamma(x|y)$ are the distributions conditional on being in the acceptance set (with densities $\upsilon(y|x)$ and $\gamma(x|y)$).[15] Substituting (16) into (17) we can write the firm fixed effect as

$$\psi(y) = \underbrace{\int_{A(y)} [w(x,y) - w_{av}(x)]\, d\Gamma(x|y)}_{=:\Psi(y)} + \int_{A(y)} \int_{B(x)} \psi(\tilde{y}) d\Upsilon(\tilde{y}|x) d\Gamma(x|y) \tag{18}$$

where $w_{av}(x)$ is the average wage of worker x, defined by $w_{av}(x) = \int_{B(x)} w(x,y) d\Upsilon(y|x)$. The fixed effect is therefore determined by a differential equation. This equation is governed by the characteristic term $\Psi(y)$ that captures the intuitive aspect that the firm fixed effect is the difference between the wage that the firm pays and the average wage that the worker receives. If the characteristic term $\Psi(y)$ is constant, it is immediate that the solution to (18) yields a constant fixed effect $\psi(y)$ that does not vary across firms.[16] That is, in this case the fixed effect does not pick up the differential desire of firms to hire better workers.

Before we proceed with the analysis, it is worth noting two points. First, for expositional simplicity we focus on first period wages. Including second period wages adds an additional term to the average wage that arises from the wages of second-period matches, but does not change the firms' fixed effects at all because the additional wage payments net out with the additional part in the workers' average wage. We provide the full analysis in the appendix. Second, our analysis is in levels rather than in log-wages. All results obtain similarly if we replaced the wage by the log-wage and the average wage by

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[14] For the specification in Proposition ?? it is easy to show that the errors are distributed symmetrically (and uniformly) around zero for the unbiased predictor and any biased estimator has more extreme errors, which increases the sum of squared errors.

[15] Let $\underline{a}(y) = \inf A(y)$ and $\bar{a}(y) = \sup A(y)$, then $\Gamma(x|y) = \Gamma(x)/[\Gamma(\bar{a}(y)) - \Gamma(\underline{a}(y))]$. Similar for $\Upsilon(y|x)$.

[16] The characteristic term $\Psi(y)$ is constant iff it is zero because the integral over the wages minus the average wage is zero. Then (18) is solved trivially when $\psi(y)$ equals zero.

the average of the log-wages.[17] Which of the formulations is most suitable is an empirical question, that is beyond the scope of our paper. The theoretical justification for focussing on the linear specification in the main text is that it leads to symmetric error terms for the simple multiplicative production function and uniform distributions that we use to derive our next results. While such a production function does induce a highly skewed wage distribution (often approximated by the log-normal), once we condition on a worker's type, that source of skewness has been picked up by w_{av}. Whether or not the variation in wages conditional on worker type is skewed or not is an empirical question. Depending on the production function the skewness can be to the left instead of to the right as implied by the log-linear formulation.

One of the benefits of our model is that we can analytically characterize the fixed effect in our environment, and we can show how it varies across firms. To show that fixed effects are not suited to analyize sorting in this model, we consider a version of our example production function:

$$f(x, y) = \alpha xy + h(x) + g(y), \tag{19}$$

where the sign of α can be either positive or negative.[18] Assume it is positive, so that the exercise is to detect the strength of sorting and not the sign. Here we dropped the exponent θ, which we can capture by considering type distributions that are not necessarily uniform.[19] It is easy to see from (12) that the acceptance set is $A(y) = [y - K, y + K]$ where $K = 2\sqrt{c/\alpha}$.

We are interested in the responsiveness of the firm fixed effect to the type of the firm. This is governed by the characteristic term $\Psi(y)$, and the fixed effect is not changing across firms if the characteristic term is constant. We are therefore interested whether higher productivity firms on average pay higher wages *once we control for the average wage that the workers in this firm are getting*. Without controlling for workers' average wages, it is trivially true that under supermodularity higher productivity firms pay higher wages. We will show that the firm fixed effect is constant under some type distributions (uniform for our example technology). This means that there will not be any correlation with the worker fixed effect.[20]

For the formal analysis of the characteristic part we focus on firms $y \in (2K, 1 - 2K)$, because their acceptance set as well as the acceptance sets of the workers they match with is in the interior of the type space. That will avoid cumbersome discussions of corner properties.[21] To see that the characteristic

---

[17]See the derivation in the appendix.

[18]To caputre (2) the term $\alpha x(1 - y)$ can be split in $-\alpha xy$ and αx, where we can attribute the latter to $h(x)$.

[19]Similarly to Footnote 4 we can capture any θ even with equation (19) through type distributions $\Gamma(x) = \sqrt[\theta]{x}$ and $\Upsilon(y) = \sqrt[\theta]{y}$ (and adjusting the additive terms approrpiately). None of our earlier results on identification relied on a uniform distribution, because we can always reformulate the problem in terms of uniform distributions (again see Footnote 4). For this part it is more convenient to work with an easier production function and a more complicated type space.

[20]The property that the firm fixed effect is zero relies on the fact that we assume that we have a large panel. If the panel is short, random variation will lead to differing fixed effects across firms. But this is just random noise, and if we had more data on each worker and firm that noise would disappear and the fixed effect would approach zero.

[21]We abstract from the types close to the edges of the type space as their matching set is constrained. Analyzing this case is not difficult, but burdensome in notation as the matching set is $A(y) = [\max\{0, y - K\}, \min\{1, y + K\}]$. Focusing on this part neglects only a small part of the type space when c and thus K is small, and has only a negligible effect on the differential equation (18) because its main effect is concentrated on the corners, which have negligible impact on intermediate types as c becomes small.

term is not necessarily responsive to the type of the firm, observe that

$$
\begin{aligned}
\Psi'(y) \;=\; & \int_{y-K}^{y+K} \frac{\partial w(x,y)}{\partial y}\gamma(x|y)dx \\
& + \left(w(y+K,y) - w_{av}(y+K)\right)\gamma(y+K|y) \\
& - \left(w(y-K,y) - w_{av}(y-K)\right)\gamma(y-K|y).
\end{aligned}
\tag{20}
$$

Call the first term $\Psi_1'(y)$ and the second and third term $\Psi_2'(y)$ so that $\Psi'(y) = \Psi_1'(y) + \Psi_2'(y)$. The first term $\Psi_1'(y)$ accounts for the wage change across those workers with which this firm type matches. The second term $\Psi_2'(y)$ reflects that the set of workers that match with a firm is slightly changing when the firm type changes.

We will focus initially on the first effect. The wage change induced by a firm is

$$
\frac{\partial w(x,y)}{\partial y} = \frac{\alpha}{2}x - \frac{\alpha}{2}y.
$$

That is, the low ability workers $x < y$ loose when the firm gets better, while the workers with relatively high ability $x > y$ gains when the firm gets better. This happens because in the first case the distance between the worker and the firm grows and the match becomes less efficient, while in the latter case the distance shrinks closer to the optimal assignment. How does this effect turn out?

For a uniform distribution the positive effect that a slightly better firm type has on the high worker types that it is willing to employ exactly offsets the negative effect on the lower worker types that it employs.[22] It is easy to see that distributions that lead to more weight on high worker types lend more weight to the positive aspect, while a distribution that lends more weight to negative worker types leads to a negative effect. We thererfore obtain positive responsiveness

$$
\Psi_1'(y) > 0 \;\; \text{if} \;\; \gamma'(x|y) > 0 \;\; \text{for all} \;\; y,
\tag{21}
$$

and likewise for opposite inequalities. For a uniform distribution $\gamma'(x|y) = 0$ and correspondingly $\Psi_1'(y) = 0$. Therefore, this effect is ambiguous and could go up or down depending on whether the economy exhibits relatively more high type workers or vice versa.

For completeness we consider also the second effect that comes through the changing matching set. This effect is more complicated, but becomes tractable when we consider the case where the firm's type density is linear. For constant conditional densities we obtain in the appendix that[23]

$$
\Psi_2'(y) < 0 \;\; \text{if} \;\; \gamma'(x|y) > 0 \;\; \text{for all} \; y,
\tag{22}
$$

and likewise with opposite inequalities. Again, for a uniform distribution $\Psi_2'(y) = 0$. So for uniform distributions (21) and (22) hold with equality, thus resulting in a constant fixed effect.[24] Similarly, when

---

[22]For the uniform distribution we have

$$
\Psi_1'(y) = \frac{1}{2K}\int_{y-K}^{y+K}\left(\frac{\alpha}{2}x - \frac{\alpha}{2}y\right)dx = \frac{1}{2K}\int_{-K}^{K}\left(\frac{\alpha}{2}(y+k) - \frac{\alpha}{2}y\right)dk = 0.
$$

[23]A constant conditional density means $\upsilon'(y|x) = r$ for some constant r. For symmetric distributions $\Gamma(\cdot) = \Upsilon(\cdot)$ this means that $\upsilon'(y|x) = \gamma'(x|y) = r$ and therefore $\Psi_2'(y) < 0$ if $r > 0$, $\Psi_2'(y) = 0$ if $r = 0$ and $\Psi_2'(y) > 0$ if $r < 0$.

[24]For this logic we ignored the boundaries of the domain (see footnote 21). Incorporating the boundaries introduces a slight positive slope of ψ on $[0, 1/2]$ and a slight negative slope on $[1/2, 1]$. This effect disappears as search costs vanish.

the distributions are uniform, the effects $\Psi_1'(y)$ and $\Psi_2'(y)$ run in opposite directions, again reducing the overall effect. It is not difficult to construct type distributions for which the overall effect is always negative or always positive for all types $y \in [2K, 1 - 2K]$.

Our main conclusion is that even in model with sorting we can analytically show that the firm fixed effect is not sensitive to the firms' true type. In finite samples we will have random noise and obviously non-trivial variation in the fixed effect, but that does not reflect the structure of the sorting and disappears as the number of periods for which we observe each worker and firm grows large. The fixed effect can be zero whether sorting is positive or negative. The ranking of firms even when we know that we have assortative matching cannot be backed out from the firm fixed-effects when the distribution is uniform, and even for non-uniform distributions the two parts that influence the fixed effect move in opposite directions. It is not difficult to see that in general the fixed effect can be non-monotone, e.g., increasing for low types and decreasing for high types, possibly including several intervals of increasing and decreasing effects. The reason that the fixed effect is not in line with the underlying type of the firm is not due to actual randomness in the workers' allocation to firms. The matching sets can be arbitrarily small so that workers focus on a very narrow band of firms with whom they are willing to match. The reason is that the increased wages for some workers are offset by decreased wages on other (worse) workers, because wages include the opportunity cost of the firm to forgoe the matching with a more appropriate worker.

4   Identifying the Strength of Sorting

We have shown that it is not possible to distinguish negative from positive sorting from wage data alone, and that in a Beckerian model of the world, fixed effects estimation not only fails to identify the sign but also the strength of sorting. This raises the question whether the strength of sorting can be identified at all from wage data. In economic terms this is the more pressing issue rather than finding the sign of sorting (positive or negative) since the welfare depends on matching the right types by avoiding inefficient mismatch, not on who these types are.

Here we develop a simple procedure to identify the degree of complementarity of the production technology $f(x, y)$. What we are after is to identify the magnitude of the cross-partial f_{xy} in absolute values. The procedure makes use of two pieces of independent information that jointly determine equilibrium:

1. The size of the *wage gap*: the difference between the highest and the lowest wage a given worker type receives. The wage gap allows us to identify the cost of search c since the marginal wage offer must make the worker indifferent between accepting and rejecting the offer, thus making the gross surplus (the wage gap) equal to the cost c.

2. The *matching range*: the fraction of jobs that are being accepted out of all jobs in the economy. This gives us independent information about the shape of the technology $f(x, y)$. To see this, observe that under strong complementarities, output changes fast when moving away from the Beckerian allocation. As a result, the output loss from non-assortative matching is large and only

a small range of matches is accepted. In contrast, under weak or no complementarities, little or no output is lost and for a given search cost a much larger range of matches is acceptable.

Suppose the wage data is generated by our model, then using the wage gap and the matching range we can estimate the extent of the complementarities, i.e., the (absolute value of the) cross-partial $|f_{xy}|$. In order to identify the model, we need repeated observations of wages of a given worker i, and and repeated observations of a given firm j. Suppose for expositional purposes that for all workers we observe T wages w_i^t paid to worker i and T wages w_j^t paid by firm j.[25] The identification works along the two steps outlined above. First, we identify the wage gap from the highest and the lowest wage. Second, we use the range of matches to identify the cross-partial.

First, we use for any agent the maximum and minimum wage to identify the *wage gap*. For an agent k (which could be either a worker or a firm) we define the maximum and minimum wage as:

$$\overline{w}_k = \max_{t \in \{1,\dots,T\}} w_k^t \quad \text{and} \quad \underline{w}_k = \min_{t \in \{1,\dots,T\}} w_k^t.$$

Indifference between accepting and rejecting the lowest paid job identifies the cost of waiting, which is given by the difference between the maximum and the minimum wage[26]

$$c_k = \overline{w}_k - \underline{w}_k.$$

Recall that in our model we have assumed that types are distributed uniformly, and we will therefore use the maximum wage also to identify the type of a worker and a firm.[27] Denote the observed cumulative distribution of maximum wages across workers by $\Omega_W(\overline{w})$. Since good workers achieve high wages, a worker's rank is given by $x_k = \Omega_W(\overline{w}_k)$. From now on we can refer to a worker by his type x and call his maximum wage $\overline{w}_x$. Similarly, let $\Omega_F(\overline{w})$ be the cumulative distribution across firms over the maximum wages paid by those firms. We attribute the following rank to a firm $y_k = \Omega_F(\overline{w}_k)$. We can now directly refer to firms by their rank y. This type correctly identifies the firms rank when we order firms by their eagerness of having a good workers (i.e., by their improvement in output from increasing the workers type). It is important to note here that this ranking reflects the firm's willingness to pay higher wages, not the firm's productivity. In fact, depending on the degree of complementarities, the rank of the most productive firms does or does not coincide with the rank of y (rank y under PAM, rank $1 - y$ under NAM).

Second, we use the *matching range* and the cost of waiting to infer the strength of the cross-partial. Denote by $\underline{y}(x)$ for each worker x the lowest firm y that it matches with, and given the uniform

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[25]Identical numbers of draws are not important, it just simplifies the exposition.

[26]We have assumed identical costs. The theory can be extended to type-dependent costs $c(x)$. See the Discussion Section.

[27]It seems natural to focus on the maximum wage because it corresponds to the Beckerian wage if the panel dimension is long enough (i.e., it is arbitrarily close to $w(k, \mu(k))$). For the current exposition we neglect the finite sample difference and assume that maximum wage in the panel exactly coincides with the theoretical optimum. We could have used the average or even the minimum wage since according to theory they all identify the same types when the panel is long. Alternatively, and in order to rule out the dominance of outliers, one can focus on the 95% interval to pin down the highest and lowest wage.

distribution of types, it determines the range of accepted matches. The loss $L(x, y)$ due to mismatch is the value that is created by pair (x, y) minus their marginal contribution (which is equal to the wage and profits) under perfect sorting

$$L(x, y) = f(x, y) - \int_0^x f_x(\tilde{x}, \tilde{x})d\tilde{x} - \int_0^y f_y(\tilde{y}, \tilde{y})d\tilde{y} - f(0, 0). \tag{23}$$

We derive in the appendix that this is equivalent to the following more concise expression

$$L(x, y) = -\int_y^x \int_{\tilde{y}}^x f_{xy}(\tilde{x}, \tilde{y})d\tilde{x}d\tilde{y} \tag{24}$$

This gives the loss when matching is indeed positive assortative.[28] Workers and firms internalize this loss in their matching decision. If $\underline{y}(x)$ is in the interior, i.e., $\underline{y}(x) > 0$, then this partner's type is determined by the indifference condition that the loss from matching equals the joint cost from waiting, i.e.,

$$-\int_{\underline{y}(x)}^x \int_{\tilde{y}}^x f_{xy}(\tilde{x}, \tilde{y})d\tilde{x}d\tilde{y} = -2c. \tag{25}$$

This is a functional equation that identifies f_{xy} evaluated at $(x, \underline{y}(x))$, for all $x \in [0, 1]$.[29]

The condition identifies the cross-partial because it compares the noise in the matching sets $(x - \underline{y}(x))$ to the noise in the wage data $(2c)$. If the wages vary substantially but matching sets are small, there must be a large loss in matching by slightly deviating from the optimal type, i.e., the cross-partial must be large.

The Parametric Example. In order to identify the functional equation, we impose additional structure using the example we outlined earlier. Under our leading example f^+ the loss due to miscoordination in absolute terms is given by

$$|L(x, y)| = \frac{|\alpha|}{2}(x^\theta - y^\theta)^2.$$

By (25) we can identify the strength of sorting via equation

$$|\alpha|(x^\theta - \underline{y}(x)^\theta)^2 = 4c$$

or equivalently

$$x = \left(2\left(c/|\alpha|\right)^{1/2} - \underline{y}(x)^\theta\right)^{1/\theta}$$

as long as $\underline{y}(x) > 0$. Parameters $|\alpha|$ and θ can be identified by the joint behavior of x and $\underline{y}(x)$. Simple non-linear regression techniques can asses these parameters if one is willing to attribute the noise in the

---

[28]If matching is negative assortative, we identified a firm type as $y = 1 - \hat{y}$ when $\hat{y}$ is the more productive firm type. Nevertheless the loss can be written exactly as in (24) for the misidentified production function $\tilde{f}(x, y) = f(x, \hat{y})$, and so the cross-partial $\tilde{f}_{xy}(x, y) = -f_{xy}(x, 1 - y)$ can be identify correctly except for the sign.

[29]Of course, given that the cross-partial is a functional equation $f_{xy}(x, y)$ which potentially can have an infinite number of parameters, makes it impossible to identify this function from finite data. We can make statements however about the average cross-partial or restricting attention to certain classes of functions, such as those with a constant cross partial $f_{xy} = \alpha$ or those that are finite polynomials. Moreover, note the standard problem that the identification in (25) only identifies the cross-partial for types that match in equilibrium (i.e. for combinations $(x, \underline{y}(x))$) and does not extend to out-of-sample combinations.

process to measurement error. The gain measured in the dollar amount that perfect matching generates in excess of complete mismatch can then be assessed simply as

$$
\begin{aligned}
\mathcal{G} &= \int_0^1 \int_0^1 |L(x,y)| dx dy \\
&= \frac{|\alpha|}{2} \int_0^1 \int_0^1 (x^\theta - y^\theta)^2 dx dy = |\alpha| \frac{\theta^2}{(2\theta + 1)(\theta + 1)^2}.
\end{aligned}
$$

Clearly one can also use the computed acceptance bounds (11) to integrate up the loss under the current mismatch compared to the frictionless optimal assignment, or to compare the current mismatch to complete randomness.

Given α and θ, we know exactly the cross-partial f_{xy} for all pairs x, y. This clearly depends on the normalization of the distribution functions (to uniform distributions) and the associated normalization of the technology f, but the dollar loss due to mismatch is invariant to the normalization. We can therefore obtain an indication of the efficiency loss of mismatch relative to the total wages for example.

5   Robustness

Our analysis of Becker's (1973) matching framework is very specific. An immediate and "valid" concern about our approach is that our finding in this specific context does not readily carry over to a more general environment. In this section we address the issue at hand in a variety of other environments. We illustrate that it is possible to construct a wage setting mechanism that does not exhibit the non-monotonicity (point 1. below), but that this is effectively a repeated static mechanism with random matching, and that it does not incorporate the dynamic trade-off with the existing match as the outside option. Such a wage setting mechanism does not incorporate the opportunity cost inherent in competitive wages as in Becker's model of sorting.

1. Repeated Static Matching. Consider the static wage determination model proposed by Abowd, Kramarz, Lengermann, and Perez-Duarte (2004). In each match between a pair (x, y) the total output is divided according to a sharing rule β in which the worker obtains $\beta f(x, y)$. This wage is monotonically increasing in y. When this is repeated, a high x worker who is matched with a low y job may choose, at a cost, to take a new draw. This will lead to some sorting since those who are most mismatched are most willing to take a new draw. Observe here that deviations are unilateral: a worker who is matched with a high y does not choose to separate, but it is the high y firm that chooses to take a new draw. As a result, inefficient separations occur as the worker is not permitted to compensate the separating firm for continuing the match. The worker would be willing to accept a lower wage in order to continue the match. This lower wage would eventually lead to the non-monotonicity.

The important aspect is that wages are independent of outside options (assumed to be zero), which makes this model essentially static. Most labor market models do not have this feature. Rather, when deciding whether to stay together or to split up, the bargaining is usually about the surplus that the pair enjoys over and above the value that each partner can ensure himself by separating. When a firm has a high value from separating, then it first gets compensated for its high outside option and only the remaining value gets split. In such a formulation if a firm is nearly indifferent between searching

for a more appropriate worker or to stay with the current worker, then it is only willing to give very little additional wage to the worker (above and beyond his compensation for not searching further). In contrast, in the repeated static matching approach, such a firm would simply search for a new worker because the share it has to give to the current worker is too large (it separates even though staying matched would be socially efficient), because the wage does not reflect the continuation payoffs.

2. Discounting. We analyzed a Beckerian model with a fixed search cost, the infinite horizon version of which is analyzed by Atakan (2006). In most search models, the cost of waiting is modeled via discounting as in Shimer and Smith (2000). We therefore now assume time discounting with factor $\beta \in (0, 1)$. We show that this lead leads to very similar conclusions: the wage profile is non-monotonic, and the fixed effect is as a result ambiguous. Waiting costs are now type-dependent as the loss is proportional to the outside option: higher types pay a higher cost from delay.

We briefly sketch the analysis with discounting under a simplified version of our production function $f^+(x, y) = xy$. When discounting replaces fixed waiting costs, a match between x and y will be formed provided the surplus exceeds the cost of delay:

$$f(x, y) - \beta \frac{x^2}{2} - \beta \frac{y^2}{2} > 0. \tag{26}$$

The matching set then is $A(y) = \left[\underline{K}y, \overline{K}y\right]$ where $\underline{K} = \beta^{-1}\left(1 - \sqrt{1 - \beta^2}\right)$ and $\overline{K} = \beta^{-1}\left(1 + \sqrt{1 - \beta^2}\right)$, and the worker's wage in a match (x, y) in the first period is given by $w(x, y)$ with derivative $\partial w/\partial y$:

$$w(x, y) = \frac{1}{2}xy + \beta \frac{x^2}{4} - \beta \frac{y^2}{4} \quad \text{and} \quad \frac{\partial w(x, y)}{\partial y} = \frac{x}{2} - \beta \frac{y}{2} \tag{27}$$

The wage is hump-shaped. The derivative is negative when $x < \beta y$. Therefore, the wage is decreasing for all worker types in $[\underline{K}y, \beta y]$. This set is non-empty for any discount factor.[30] While in this setting identification might be possible, the difference between wages of positive and negative assorted production functions is of order $(1 - \beta)$ and therefore hard to detect when agents are patient.[31]

Figure 4 illustrates the wage pattern as a function of the distance k between the worker and the firm. The wages at the boundaries reflect the value from waiting and therefore necessarily are the same. In the middle the distribution is single-peaked, only that the peak is shifted to higher wages compared to the previous section. Still roughly for half of the acceptance set the wage is decreasing in y and falls to the value of waiting at the boundary. Again this induces ambiguous firm fixed effects. The change in the characteristic term in (18) can be compute in analogy to (20) as:[32]

$$
\begin{aligned}
\Psi'(y) \quad = \quad & \int_{\underline{K}y}^{\overline{K}y} \left(\frac{1}{2}x - \frac{1}{2}\beta y \right) \gamma(x|y)dx \\
& + \overline{K} \left(\beta \frac{\overline{K}^2 y^2}{2} - w_{av}(\overline{K}y) \right) \gamma(\overline{K}y|y) - \underline{K} \left(\beta \frac{\underline{K}^2 y^2}{2} - w_{av}(\underline{K}y) \right) \gamma(\underline{K}y|y).
\end{aligned}
$$

---

[30]It is straightforward to show that $\underline{K} = \beta^{-1}\left(1 - \sqrt{1 - \beta^2}\right) y < \beta y$ if and only if $1 - \beta^2 > (1 - \beta^2)^2$, which is true for all $\beta \in (0, 1)$.

[31]Under the submodular specification $f^-(x, y) = (1 - y)x + y$ the wage will be as in (27) when we replace y by its transform $\tilde{y} = 1 - y$, plus an added term $\frac{1}{2}(1 - \beta)(1 - \tilde{y})$.

[32]Now, the area of integration is adjusted to $A(y) = \left[\underline{K}y, \overline{K}y\right]$ and the conditional distribution to $\Gamma(x|y) = \Gamma(x)/[\Gamma(\overline{K}y) - \Gamma(\underline{K}y)]$. The second term in expression (??) only arises for $\underline{K}y < 1$, otherwise the upper bound is 1 and is not affected by a change in y.
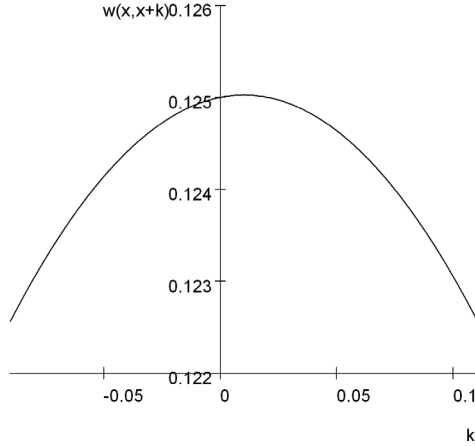
Figure 4: First period wages for under mismatch with a type that is k away from the bliss point. [Graph for $x = .5, \beta = .98$]

This effect can be positive, negative, or zero, depending on the exact nature of the type density. Again we believe that an approach based on the bounds of the acceptance set relative to the entire type space similar to Section 4 will be useful in considering the strength of sorting.

3. General Type-dependent Search Costs or Arrival Rates. As is apparent from the case with discounting as in Shimer and Smith (2000), the non-monotonicity of the wage schedule continues to hold when costs are type-dependent and asymmetric. Then the condition for accepting a match is:

$$f(x, y) - (w^\star(x) + \pi^\star(y) - c(x) - c(y)) \geq 0.$$

where in the case of discounting, $c(x) = (1 - \beta)w^\star(x)$ and likewise for $c(y)$. A general type-dependent search cost in our example implies a wage

$$w(x, y) = \frac{1}{2}xy + \frac{1}{4}x^2 - \frac{1}{4}y^2 - \frac{1}{2}c(x) + \frac{1}{2}c(y)$$

and will in general continue to induce a non-monotonic wage schedule, as $\partial w / \partial y = 1/2x - 1/2y + c'(y)$.

The general type-dependent search cost is also isomorphic to the case of type-dependent arrival rates. This captures the notion that different types may have more difficulty in finding a job, for example it may be harder to find a job for a high skilled CEO than a low skilled burger flipper. To see that this is equivalent to the case of general search cost, observe that we can model arrival rates via a probability $\alpha(x)$ of entering the second period of our model. Then the condition is

$$f(x, y) - (\alpha(x)\beta w^\star(x) + \alpha(y)\beta \pi^\star(y)) \geq 0.$$

where as before, the equivalent $c(x) = (1 - \alpha(x)\beta)w^\star(x)$.

4. Directed Search. In the directed search model of Shimer (2005) with ex post screening, complementarities lead to sorting. The firms offer strategies are monotonic, in the sense that a higher

worker type obtains higher wages from better firms. Interestingly, in that model a non-monotonicity may appear on the workers side, since a higher worker type may obtain a lower wage at a given firm. Alternatively, if workers were to auction off their labor, then the equilibrium wages offered would be again non-linear in firm type.[33]

5. On-the-job Search. On-the-Job-Search (OJS) is another likely candidate for identifying sorting using equilibrium mismatch. Bagger and Lentz (2008), Lise, Meghir and Robin (2008) and Lopes de Melo (2008) consider a sorting model with on-the-job search (as in Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006)). As long as a job is scarce, matched pairs face a trade-off between matching early and waiting for the appropriate types.[34] This arises because in nearly all existing models a current match allows only the workers to search further but precludes the firms from further matches, and thus introduces an opportunity cost for inappropriate matches. Even if both sides can search such an opportunity cost remains as long as arrival rates in a match are lower than for unmatched agents. In such cases on-the-job search will affect the exact nature of the equilibrium wages, but high firms will still only pay relatively low wages to low workers to compensate them for their lost continuation value without that worker.

6   Discussion and Extensions

In this paper we pursue two goals. First, we use the most well-known model of sorting by Becker (1973) to gain insights into the wage setting process in a competitive environment, which serves as a natural benchmark. We extend that model in the smallest possible way to allow for mismatch while retaining the basic idea underlying the assortative matching model. This allows us to provide analytical expressions for the mismatched wages in the model, to characterize their bliss point property, and to provide an explicit version of a fixed effects method used in the empirical literature. We show that the latter is neither well-suited to identify the sign nor the strength of sorting. Identification of the sign of sorting is in general impossible because firms pay wages based on the gain they have from employing a higher worker, not because they themselves are productive. Even under positive sorting the fixed effects approach is not able to identify the strength of sorting because wages are non-monotone in firm type. This non-monotonicity is at odds with the basic fixed-effects idea, and we show that the net effect may be zero.

Second, we propose to abandon the attempt to identify from wage data the sign of sorting. The mere fact that wages are determined mainly by the need for having a better worker (which is based on the cross-partial and not the first derivatives) makes such identification difficult. Under submodularity it is the low productivity firms that especially need good workers to increase their (in terms of levels) meager profits, while under submodularity it is the productive firms that need more productive workers most. In both cases the firms that need the productive workers most have an incentive to pay high wages, which makes identification without profit (per job) data difficult. Yet in economic terms the sign of sorting may be less important than the gain that is achieved by sorting workers into the "right"

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[33]We are grateful to Robert Shimer for pointing this out to us.

[34]In Bagger and Lentz (2008) jobs are not scarce since firms can open as many jobs as they want. The sorting effect in their model derives from differences in the intensity with which workers search for a new job.

job. We show that some information about this gain can be identified from wage data. We propose a specific method of backing out this strength along the equilibrium path. The identification comes from determining some notion of the size of the set of firms with which a worker matches. If a worker is only willing to match with a small fraction of firms, for a given level of frictions (which we can identify from the data) the complementarities must be large. Similarly, when a worker is willing to match with many firm types the complementarities must be weak. This gives a well-defined notion of the dollar-value of the gain from sorting in the market.

Our method may well not be the only one that identifies the strength of sorting. Since existing attempts have mainly considered the sign of sorting, this area is not well-developed. We believe that other methods that compare the noise in the *accepted* matches of a worker (i.e., the range of accepted firm types or the range of accepted wages) with the total noise *in the market as a whole* are promising in shedding light on this issue. Similarly, one can look at the problem from the firm side. When within-firm variation of worker types (or their salaries – depending on the model) is low relative to the overall variation in the data, for given level of frictions complementarities must be large for agents to focus on a narrow band of matches. For example, Lopes de Melo (2008) considers the within-firm correlation of wages as a measure. Comparisons with the variation overall might capture something about the strength of sorting. Evaluating the strength of sorting this way is important in order to assess the welfare-consequences of matching agents better. The main difficulty is to distinguish the cause of the relative noise levels. It could be that frictions are high and therefore workers accept nearly all matches, or complementarities are low. The challenge is to propose procedures that separate the source of frictions from the complementarity, and the procedure in the previous section presents one approach to do this.

We now briefly discuss some of the other issues that may be of importance in identifying sorting.

Using Data on Profits: the Attribution Problem. One obvious observation is that with data on profits we can identify both the strength and the sign of sorting. And while there are good data on firm profits, the problem is that there are no data on job profits. In multi-worker firms, we need to attribute the share of each worker's contribution to the overall firm profits. Even in the simplest economy we need to decide what the contribution of very different individuals (CEO, accountant, and secretary) is to the firm profit. Since this decomposition seems difficult across occupations[35], we propose to focus on the economically important and manageable problem of the identifying the strength of sorting.

Being aware of the shortcomings of the wage data in fixed effects estimates, Mendes, van den Berg, and Lindeboom (2007) use productivity data instead. They choose *average* firm-specific productivity to attribute output from the firm to an individual worker. They find that average firm-specific productivity and worker skill exhibit strong positive sorting.

More General Technologies. Above, we have shown that the *strength* of the sorting can be identified, but not the sign. This suggests that our analysis extends to an even broader class of preferences.

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[35]In a simple matching model of the firm, Eeckhout and Pinheiro (2008) show that only under very specific conditions, namely homotheticity in the production technology, wage ratios of different skills within a firm will be the same as those across firms. In all other cases, the wage share of a given skill is different in different firms, and as a result simply attributing profits to jobs proportional to wages will be biased.

Suppose output is maximized when "similar" agents match, then there is no ranking of better jobs, but we can still identify how strong the complementarity is between workers with our approach. Observe that this does include more realistic cases of production technologies where types are multi-dimensional. All the information that we use to identify the sorting effect is embedded in the wages, thus giving us a monetary value (and therefore one-dimensional order) of sorting.

7   Concluding Remarks

We argue that identifying the sign of sorting from wage data alone is difficult, if not impossible. The main reason is that the wages reflect at least in part the marginal contribution to the value that the firm generates, and it can be either the more productive or the less productive firms that have a higher marginal benefit from employing a better worker. The empirical question whether or not there is evidence of sorting remains to be answered. In many settings we expect more able workers to have a higher marginal contribution to more productive firms, but in some industries more productive firms have invested in automatization that allows workers with lower skills to perform the high productivity jobs (e.g., retail trade). We argue that it is nonetheless possible to identify the strength of sorting, though the commonly used fixed effect methods are not suitable for the task. With estimates of the strength of sorting in hand, the efficiency loss of mismatch and the benefits of market interventions can be assessed.

8 Appendix

Fixed-Effect Decomposition in Equation (15): Our residual is given by

$$\varepsilon_{xy} = w(x,y) - \delta(x) - \psi(y)$$

For a given firm y, the average residual across the workers it matches with in the first period is zero, since

$$
\int_{A(y)} \varepsilon_{xy} d\Gamma(x|y) = \int_{A(y)} \left(w(x,y) - \delta(x) - \int_{A(y)} [w(\tilde{x},y) - \delta(\tilde{x})] d\Gamma(\tilde{x}|y) \right) d\Gamma(x|y) \qquad (28)
$$

$$
= \int_{A(y)} [w(x,y) - \delta(x)] d\Gamma(x|y) - \int_{A(y)} [w(\tilde{x},y) - \delta(\tilde{x})] d\Gamma(\tilde{x}|y) = 0,
$$

where in the second line the double integral disappears because the interior integral is constant with respect to the argument of integration of the outer integral and the $\int_{A(y)} d\Gamma(x|y) = 1$ since $\Gamma(x|y)$ is a cumulative distribution function on $A(y)$. Similarly, for a given worker x the average residual is zero across the firms that he matches with in the first period, since

$$
\int_{B(x)} \varepsilon_{xy} d\Upsilon(y|x) = \int_{B(x)} \left(w(x,y) - \int_{B(x)} [w(x,\tilde{y}) - \psi(\tilde{y})] d\Upsilon(\tilde{y}|x) - \psi(y) \right) d\Upsilon(y|x) \qquad (29)
$$

$$
= \int_{B(x)} [w(x,y) - \psi(y)] d\Upsilon(y|x) - \int_{B(x)} [w(x,\tilde{y}) - \psi(\tilde{y})] d\Upsilon(\tilde{y}|x) = 0
$$

It is obvious that all transformations remain true if we use log-wages, as long as we define $w_{av}(x)$ as the average over the log-wages.

Finally, observe that our analysis is not affected by the fact that we concentrate on first period wages. Assume we look at wages across all matches. Equation (15) remains identical, but we use more data (i.e. also the second-period matches) to obtain the fixed effects. To derive the fixed effects in this case, let $\underline{a}(y) = \inf A(y), \underline{b}(x) = \inf B(x)$, $\bar{a}(y) = \sup A(y)$ and $\bar{b}(x) = \sup B(x)$ denote the boundaries of the acceptance sets for the firms and workers. Worker x and firm y matches with probability $\Upsilon(\bar{b}(x)) - \Upsilon(\underline{b}(x))$ and $\Gamma(\bar{a}(y)) - \Gamma(\underline{a}(y))$ in the first period, and with complementary probability they match in the second period.

In this case the overall fixed effects are

$$
\hat{\delta}(x) = \left[\Upsilon(\bar{b}(x)) - \Upsilon(\underline{b}(x)) \right] \int_{B(x)} [w(x,y) - \psi(y)] d\Upsilon(y|x) + \left[1 - \Upsilon(\bar{b}(x)) + \Upsilon(\underline{b}(x)) \right] [w(x,\mu(x)) - \psi(\mu(x))] \quad (30)
$$

$$
\hat{\psi}(y) = \left[\Gamma(\bar{a}(y)) - \Gamma(\underline{a}(y)) \right] \int_{A(y)} [w(x,y) - \delta(x)] d\Gamma(x|y) + \left[1 - \Gamma(\bar{a}(y)) + \Gamma(\underline{a}(y)) \right] \left[w(\mu^{-1}(y),y) - \delta(\mu^{-1}(y)) \right] \quad (31)
$$

Due to our symmetry assumptions on Υ and Γ, substituting (30) into (31) yields exactly equation (18) in the main text that characterizes the firm fixed effect. Therefore, the firm fixed effect is not changed. The intuitive reason is that the firm fixed effect is governed by the *additional* wage beyond the workers average (and in second period matches this is exactly zero), and the same conclusions that we obtain for the first period wages carry over to the entire model.

Derivation of (22): Observe that both $w(y+K,y)$ and $w(y-K,y)$ are wages for workers that are exactly indifferent between matching now and not matching. This indifference implies $w(y+K,y) = h(y+K) + \frac{\alpha(y+K)^2}{2} - c$ and $w(y-K,y) = h(y-K) + \frac{\alpha(y-K)^2}{2} - c$. Since the average wage that a worker gets when matching in the first stage is better than not matching, the difference between these wages and the average wage is negative. Using these expressions and the fact that we can write the average wage as

$$w_{av}(x) = \int_{x-K}^{x+K} w(x,y)d\Upsilon(y|x) = h(x) + \frac{\alpha}{2}x^2 - \frac{\alpha}{4}\int_{-K}^{K} k^2 v(x+k|x)dk \tag{32}$$

yields

$$\begin{aligned}
\psi_2'(y) &= -\left[c - \frac{\alpha}{4}\int_{-K}^{K} k^2 v(y+K+k|y+K)dk\right]\gamma(y+K|y) \\
&\quad + \left[c - \frac{\alpha}{4}\int_{-K}^{K} k^2 v(y-K+k|y-K)dk\right]\gamma(y-K|y).
\end{aligned} \tag{33}$$

This effect depends on y only through the effect on the distribution. Therefore, again this effect is zero under a uniform distribution. For other distributions the effect is ambiguous because it relies on the density at the endpoints as well as on the integral over the density. In the special case where the derivative of the conditional density $\partial v(y|x)/\partial y = r$ is constant the density is linear, and so are the conditional densities. In such a case symmetry around zero of k^2 ensures that $-\frac{1}{4}\int_{-K}^{K} k^2 v(Y+k|Y)dk$ is independent of Y and the terms in square brackets are identical, which directly leads to the inequalities in (22).

Derivation of (24): Equation (24) can be expanded to

$$\begin{aligned}
L(x,y) &= \int_0^y f_y(x,\tilde{y})d\tilde{y} + f(x,0) - \int_0^x f_x(\tilde{x},\tilde{x})d\tilde{x} - \int_0^y f_y(\tilde{y},\tilde{y})d\tilde{y} - f(0,0) \\
&= \int_0^y \int_0^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} + \int_0^y f_y(0,\tilde{y})d\tilde{y} + f(x,0) \\
&\quad - \int_0^x \int_0^{\tilde{x}} f_{xy}(\tilde{x},\tilde{y})d\tilde{y}d\tilde{x} - \int_0^x f_x(\tilde{x},0)d\tilde{x} - \int_0^y \int_0^{\tilde{y}} f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} - \int_0^y f_y(\tilde{y},0)d\tilde{y} - f(0,0) \\
&= \int_0^y \int_{\tilde{y}}^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} - \int_0^x \int_0^{\tilde{x}} f_{xy}(\tilde{x},\tilde{y})d\tilde{y}d\tilde{x}
\end{aligned}$$

Let $x \geq y$. (The derivation under the opposite follows analogous steps.) Then we have

$$L(x,y) = \int_0^y \int_{\tilde{y}}^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} - \int_0^x \int_0^{\tilde{x}} f_{xy}(\tilde{x},\tilde{y})d\tilde{y}d\tilde{x} \tag{34}$$

Note that $\int_0^x \int_0^{\tilde{x}} f_{xy}(\tilde{x},\tilde{y})d\tilde{y}d\tilde{x}$ integrates for each $\tilde{x}$ over all $\tilde{y} \leq \tilde{x}$. Similarly, one can for each $\tilde{y}$ integrate over all $\tilde{y} \geq \tilde{x}$. That is

$$\int_0^x \int_0^{\tilde{x}} f_{xy}(\tilde{x},\tilde{y})d\tilde{y}d\tilde{x} = \int_0^x \int_{\tilde{y}}^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y}.$$

Therefore, (34) becomes

$$L(x,y) = \int_0^y \int_{\tilde{y}}^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} - \int_0^x \int_{\tilde{y}}^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} = -\int_y^x \int_{\tilde{y}}^x f_{xy}(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y}.$$

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